KNOWLEDGE, INFORMATION, AND EXPECTATIONS IN

MODERN MACROECONOMICS

IN HONOR OF EDMUND S. PHELPS

EDITED BY

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— 10 —

Endogenous Fluctuations and the Role of Monetary Policy

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1. INTRODUCTION

Why is monetary policy a desirable social tool and why is public action in this area justified? The controversial nature of these questions arises from the fact that any answers are linked to two related questions: Why does a competitive market economy experience excess fluctuations and why is money nonneutral? Hence, a theory of stabilizing monetary policy has to provide a *unified* explanation as to why our economy experiences fluctuations and why monetary policy can have an impact on these fluctuations.

In a sequence of earlier papers we have argued that most volatility in financial markets is caused by the beliefs of agents (see Kurz and Schneider, 1996; Kurz 1997a,b; Kurz and Beltratti, 1997; Kurz and Motolese, 2001). Using the theory of rational belief equilibrium (RBE) (see Kurz, 1994, 1996, 1997a), we introduced a unified model which explains, simultaneously, a list of financial phenomena regarded as "anomalies." The model's key feature is the heterogeneity of an agent's beliefs. At any time an agent may be a "bull" who is rational but optimistic about future capital gains or a "bear" who is rational but pessimistic about such gains. Phenomena such as the equity premium puzzle are then explained by the fact that pessimistic "bears" who aim to avoid capital losses drive interest rates low and the equity premium high (for a unified treatment see Kurz and Motolese, 2001). The RBE theory was used by Kurz (1997c) to explain the forward discount bias in foreign exchange markets; by Garmaise (1998) to explain the capital structure of firms, and by Wu and Guo (2001) to study speculation and trading volume in asset markets. This chapter initiates our application of the RBE theory to the study

of endogenous fluctuations in a monetary economy and to an examination of the implied role of monetary policy.

The idea that diverse expectations are important in an equilibrium analysis is not new to economics. Diverse beliefs in financial markets are a central component of Thornton's (1802) view of paper money and financial markets. Expectations are often mentioned in Keynes (1936), although he never developed a formal theory of individual beliefs. Market expectations are central to "cumulative movements" in Pigou (see Pigou, 1941, chap. 6) and expectations are basic to the process of deviations from a stationary equilibrium in the Swedish school (e.g., see Myrdal's view of money in Myrdal, 1939, chap. 3). Also, the concept of "subjective values" based on diverse expectations is a cornerstone of Lindahl's (1939) theory of money and capital. Finally, diverse expectations are generic to an Arrow-Debreu or a Radner equilibrium.

As this work is dedicated to Ned Phelps we observe that he often stressed the importance of expectations. In accord with a Bayesian perspective he saw expectations as subjective models expressing the agent's interpretation of information and needing a dynamic updating that would lead to some concept of equilibrium. Although attracted by the simplicity of the single-belief model, he realized its lack of realism and, more important, its failure to capture the component of volatility induced by the interaction of heterogeneous expectations. More specifically, in evaluating the impact of rational expectations, Frydman and Phelps (1983) stress the importance of diverse beliefs. They justify their position on the ground that any theory with uniform market beliefs of agents is fundamentally nonrobust, saying: "But once the theoretical door is opened to one or more hypotheses of optimality in the expectations formation of the individual agents, the implied behavior of the (otherwise identical) model is often found to be wrenched into directions far from the behavior implied by the rational expectations hypothesis. In short, Pandora's box of disequilibrium behavior is opened up" (p. 26). The theory of rational beliefs (RB) provides an analytical framework and a vocabulary for these ideas. It shows that the interaction of beliefs acts as a propagation mechanism generating volatility endogenously. The component of social risk generated by the distribution of beliefs is called "endogenous uncertainty." Although RB generalize the concept of rational expectations, the RBE theory reveals that, as Frydman and Phelps (1983) conjectured, an economy with diverse beliefs behaves in a drastically different way from an economy with a single, uniform belief.

1.1. On the Diversity of Rational Beliefs

Table 1 presents, as an example, forecasts of GNP growth and inflation made in January 1991 for all of 1991 by participants in the *Blue Chip Economic Indicators*. About half of the forecasters predicted at that time that 1991 would be a recession year and the other half disagreed. The actual growth rate in 1991 was −0.5 percent and the inflation rate 3.6 percent. Now, placing ourselves in January 1991, suppose we make a stationary econometric forecast of GNP growth without nonstationary judgments about the unique conditions prevailing in 1991. We used a model of Stock and Watson (1999a,b, 2001), estimated by employing a combination

Table 1. Blue Chip Forecasts of GNP Growth and Inflation for 1991

Company	Forecasted percent change	
	Real GNP	*GNP price deflator*
Sears Roebuck & Co.	1.6H	4.2
Amhold & S. Bleichroeder	1.2	4.8
Prudential Bache	1.2	3.3L
Chicago Corporation	1.1	4.1
Bostian Economic Research	1.0	4.0
Fairmodel	1.0	3.7
Cahners Economics	0.9	4.3
Wayne Hummer & Co.—Chicago	0.8	4.3
National City Bank of Cleveland	0.7	4.6
Inforum—University of Maryland	0.7	3.8
CRT Government Securities	0.6	4.0
Dun & Bradstreet	0.6	4.0
Conference Board	0.5	4.7
Econoclast	0.5	4.0
First National Bank of Chicago	0.5	3.8
University of Michigan M.Q.E.M.	0.4	4.7
Manufacturers National Bank—Detroit	0.3	4.5
Turning Points (Micrometrics)	0.2	4.3
Brown-Brothers Harriman	0.2	4.0
Dean Witter Reynolds, Inc.	0.1	4.0
LaSalle National Bank	0.1	3.6
Northern Trust Company	0.0	4.3
Evans Economics	0.0	4.0
Morris Cohen & Associates	−0.1	5.0H
Prudential Insurance Co.	−0.1	4.5
Chrysler Corporation	−0.1	4.1
Econoviews International Inc.	−0.1	3.9
U.S. Trust Co.	−0.2	4.3
Reeder Associates (Charles)	−0.3	4.9
Siff, Oakley, Marks Inc.	−0.3	4.8
Morgan Stanley & Co.	−0.3	4.7
Eggert Economic Enterprises, Inc.	−0.3	3.9
CoreStates Financial Corp.	−0.4	4.3
Mortgage Bankers Association of America	−0.4	4.3
Bank of America	−0.4	3.6
E. I. Du Pont de Nemours & Co.	−0.5	4.8
National Association of Home Builders	−0.5	4.5
Metropolitan Life Insurance Co.	−0.5	4.5
Ford Motor Company	−0.6	4.6
Chase Manhattan Bank	−0.6	4.0
U.S. Chamber of Commerce	−0.7	5.0H
Manufacturers Hanover Trust Co.	−0.7	4.4
Bankers Trust Co.	−0.7	4.4
Laurence H. Meyer & Assoc.	−0.7	4.0
Security Pacific National Bank	−0.7	4.0
PNC Financial Corp.	−0.9	4.3
UCLA Business Forecast	−0.9	4.2
Merrill Lynch	−1.1	4.4
Georgia State University	−1.1	3.6
Equitable Life Assurance	−1.2	4.7
Morgan Guaranty Trust Co.	−1.2	3.8
Shawmut National Corp.	−1.3L	4.0

of diffusion indexes and averaged bivariate VAR forecasts and utilizing a large number of U.S. time series. All nonjudgmental stationary forecasts of GNP turned out to be *higher than most of the private forecasts*.

Figure 1 presents the distribution of private forecasts of GDP growth rates in 1990–2001. These forecasts were made in each quarter for GDP growth over the full year *following the year of the forecast*. Hence, in each quarter of a given year the four forecasts were made for the *same year*. For example, in March, June, September, and December of 1994 individual forecasts are for the full year 1995. Figure 1 exhibits the fifth percentile and the ninety-fifth percentile of the forecast distribution in each quarter and the horizontal bars show the realization of GDP growth a year later. For example, the bar in 1999 exhibits the growth rate realized in 2000 and forecasted in each of the four quarters of 1999. The stationary forecasts (not shown) are narrowly distributed just below 3 percent: In some periods private forecasts are above and in other periods they are below the stationary forecasts.

The empirical record thus reveals wide fluctuations of individual forecasts of variables about which there is no private information. Moreover, individual forecasts fluctuate over time around the stationary forecasts, showing no sign of convergence to these nondiscretionary stationary forecasts. So, how does the theory of rational beliefs explain these facts?

An account of the RB theory can be found in Kurz (1997a) or Kurz and Motolese (2001). Here we explain briefly that the RB theory *assumes* that agents do not know the true probability underlying the equilibrium process but that they have a great deal of past data about the observable variables in the economy. Using past data agents compute the empirical distribution and construct from it a probability measure over infinite sequences of observable variables. It can be
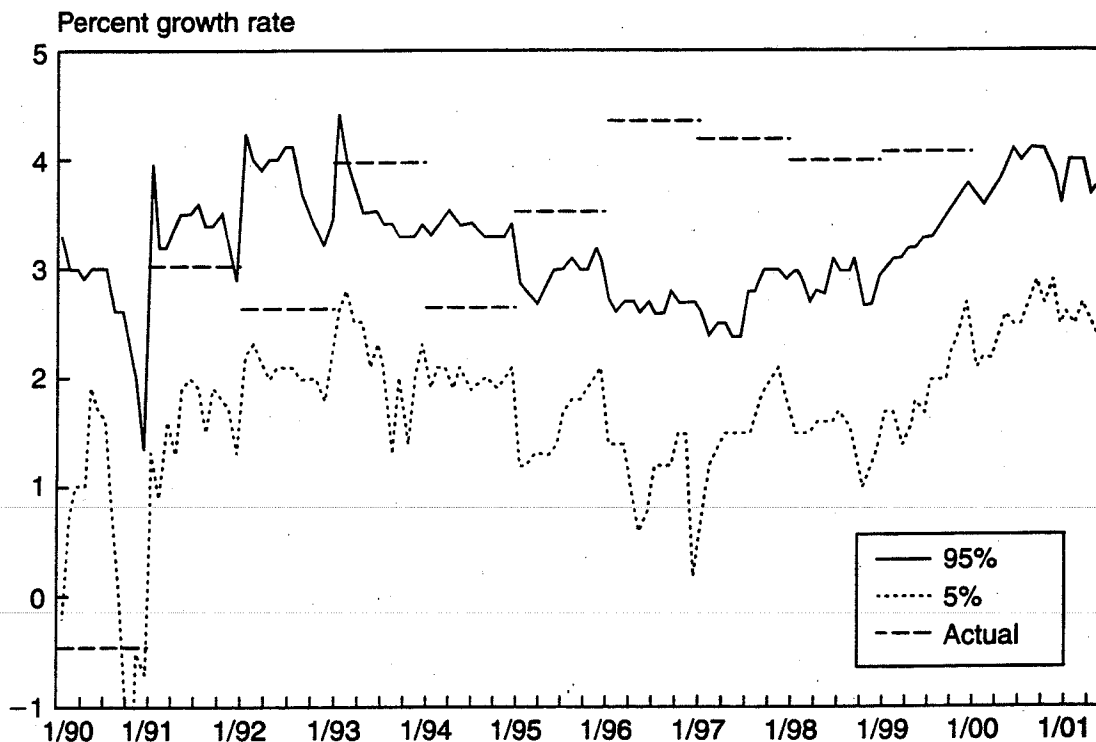

Figure 1. Distribution of private forecasts of GDP growth.

shown that this probability is stationary and hence *agents do not disagree about the stationary forecasts*. Since the true equilibrium process is not known and is likely to be nonstationary, agents who believe the process to be nonstationary construct their own subjective probability models. A *rational belief* is then a probability over infinite sequences of observables, perhaps nonstationary, that *cannot be contradicted by the empirical evidence*. More specifically, if one uses an RB to simulate the data of the economy it will reproduce the known empirical distribution of equilibrium observables. An RB may, at times, exhibit "optimism" or "pessimism" relative to the stationary forecasts, but it would be irrational to forecast *at all dates* above or below the stationary forecasts. That is, rationality requires the RB forecasts to be compatible with the stationary forecasts: The time average of the conditional forecasts must be equal to the conditional stationary forecasts. It is a theorem that nonstationary rational beliefs act as in the data: Conditional upon given observables, the forecasts fluctuate over time around the stationary forecast with no tendency toward convergence. In short, the RB theory proposes that the forecasts in Table 1 and Figure 1 are made by agents who do not believe that the dynamics of the economy is stationary. They use their own models, combining past data with subjective assessment of the unique conditions prevailing in the market at each date.

The purpose of this chapter is to study the implications of RBE for the dynamics of monetary economies, and hence we assume that agents hold rational beliefs rather than rational expectations. We examine to what degree standard results of monetary analysis can be generated by an RBE in comparison with assumptions employed in other monetary models. It would thus be helpful to briefly clarify our assumptions in relation to other current work on economic fluctuations and monetary economics.

1.2. Our Assumptions in Relation to the Literature

i. Technological Shocks and Investments. Traditional real business cycle (RBC) models used the Solow residual as a measure of technological shocks. However, empirical evidence has revealed that this measure depends upon endogenous variables such as capacity utilization (e.g., Greenwood et al., 1988; Burnside et al., 1995; Basu, 1996). Computational limitations preclude us from introducing endogenous capacity utilization and we focus only on the sign and size of productivity shocks. We agree with the critique of most writers (e.g., Summers, 1986; Eichenbaum, 1991) that exogenous technological shocks constitute a small fraction of the Solow residual and negative technological shocks are unjustified.[1] We thus conclude that the observed data of economic fluctuations cannot be explained by an exogenous process of nonnegative productivity shocks with small variance. For examples of alternative approaches within the RBC tradition see Wen (1998a,b) and King and Rebelo (1999).

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1. In a recent paper Cole and Ohanian (2000) argue that the Great Depression resulted from a cumulative 15 percent negative technological shock to the U.S. economy during 1929–1932. We cannot support such a conclusion.

We assume *a very small variance* of the exogenous technological shocks but explicitly incorporate an investment goods sector, which transforms consumption goods into capital goods ("machines"), to model the uncertainty associated with the outcome of new investments. The output of the investment sector is stochastic, so investment decisions must be based on the assessment of this risk. The investment sector itself is very simple, consisting of two projects that vary in productivity depending upon specified random shocks. We then investigate the impact of the distribution of beliefs on aggregate fluctuations of investments, output, and prices.

ii. Asymmetric Information. In a rational expectations equilibrium (REE) the study of monetary policy focused, at the early stages, on the effects of informational asymmetries. Lucas (1972) argued that money has real effect because people confuse changes in price level with changes in relative prices. By implication, monetary policy has a real effect only when it is unanticipated. Empirical evidence has not supported this conclusion (see, e.g., Mishkin, 1982), revealing that both anticipated as well as unanticipated changes in money have real effects. In an RBE agents hold different conditional probabilities and arrive at different conclusions when they condition on current information. That is, *agents interpret current information differently*. Hence, even fully observed monetary shocks might be interpreted differently by agents, and money nonneutrality is implied (for earlier work, see Motolese, 2000, 2001). Focusing here on the effects of beliefs we thus assume in this chapter *that there is no asymmetric information.*

iii. Sticky Prices, Monopolistic Competition, and Credit. The *dynamic new Keynesian* theory (DNK) has developed an integrated view of monetary equilibrium, built upon two basic assumptions: (1) The market consists of monopolistically competitive firms that are price-setters, and (2) prices are "sticky" owing to the existence of long-term contracts (e.g., Taylor, 1980; Calvo, 1983; Yun, 1996; Clarida et al., 1999; Rotemberg and Woodford, 1999; Woodford, 1999, 2000, 2001a,b). Many authors work with Calvo's (1983) idealization, in which at any date only a fraction of firms are "allowed" to change prices. In such an economy the object of monetary policy is to restore efficiency by countering the negative effect of price rigidity. Although the DNK offers a unified perspective of monetary policy, the model of sticky prices explains very little of the observed volatility in our economy, and we shall thus not adopt any of the assumptions of the DNK. Our RBE is an equilibrium of an economy that is *fully competitive* in which *prices are fully flexible*. In doing so we are not rejecting the proposition that some money nonneutrality is due to wage or price rigidity, and such rigidity has important policy implications. Moreover, some recent versions of the DNK model propose that an important effect of monetary policy operates through its impact on the availability of credit to borrowing firms. Bernanke and Gertler (1989), Carlstrom and Fuerst (1997), and Bernanke et al. (1999) argue that fluctuations in the return to capital and in asset prices alter the collateral available to borrowers, generating an amplification that they call a "financial accelerator." Although we agree that credit is an important component of monetary analysis, the financial accelerator

was developed under strong assumptions. We hope to incorporate in our model a credit amplification mechanism that operates under weaker assumptions.

iv. The Rationale for Monetary Policy. Most optimal policy models do not explain why a central bank should follow a stabilization policy. They are typically formulated as reduced-form systems of equations representing the private sector. The central bank then selects *directly* optimal inflation and GNP growth rates so as to minimize a quadratic objective (e.g., Taylor, 1993, 2000). The *implicit* justification for stabilization policy is the view that the private allocation is not Pareto optimal, reflected by fluctuations in the growth rate of GNP and the rate of inflation around their targets. The targets themselves are treated as Pareto-optimal states. Our objectives in this chapter are very limited. We do not characterize optimal monetary policy and only use the "policy" of exogenous variations in money supply. Our modest aim is to study the impact of diverse beliefs in an RBE on the dynamics of a monetary economy. We demonstrate that the distribution of beliefs causes endogenous amplification of fluctuations and argue that Pareto optimality is not an adequate criterion for monetary policy. A preliminary view of stabilization policy is discussed in Section 4.

We thus formulate a monetary model with two infinitely lived agents and study its volatility. We use numerical simulations to demonstrate the theoretical properties of the model but *do not attempt an accurate calibration.* To sum up our conclusions we note first that our investment model is simplistic, incorporating compromises to facilitate computations. Consequently some results are counter-factual and hence the chapter offers only qualitative and conceptual results. With these qualifications stated, this chapter proposes three basic perspectives:

1. The RBE paradigm offers an integrated theory of real and financial volatility with a high volume of trade. Most volatility in an RBE is induced endogenously through the beliefs of agents.
2. Although our RBE assumes *fully competitive* markets in which *prices are fully flexible,* the diverse expectations of agents can explain most of the familiar features of monetary equilibria. This includes money non-neutrality, Phillips curve, and impulse response functions with respect to monetary shocks.
3. Agents with diverse but inconsistent beliefs may induce socially undesirable excess fluctuations even when the allocation is ex ante Pareto-optimal. Central bank policy should aim to reduce the endogenous component of this volatility.

2. THE ECONOMIC ENVIRONMENT

The model economy has four traded goods: a consumption good, a capital good, labor services, and money. Agents can buy existing capital goods on the open market but new capital goods are produced by two alternative activities whose output depends upon a random shock that affects only the investments sector. The decision of how much installed capital goods to buy on the open market and how

much to invest in new projects depends upon the beliefs of the agents and upon the price of capital goods. Investments are irreversible: Once produced, capital goods cannot be turned back into consumption goods although they depreciate with use. There are two infinitely lived agents with utility over consumption, labor services, and real money holding. These agents make all the intertemporal decisions in this economy. The income of agents consists of labor income and the income they receive from assets that they trade in competitive markets. The first asset is an ownership unit of real capital goods employed by the firms in the economy. Aggregate supply of such units equals the number of units of capital in the economy. At each date a unit of capital pays a risky dividend and has a risky return consisting of dividends and capital gains. The second asset is a one-period bill that pays a riskless *nominal* return and has a zero net supply. The third asset is fiat money issued by the central bank. Under the money supply policy studied in this chapter a *random* change in the money supply results in a random change in the money holding of an agent. To avoid issues related to public budget constraint, the expected growth rate of money equals the expected growth of output and the model exhibits zero long-run inflation.

Competitive firms in this economy are myopic in outlook. At each date they hire labor services and rent capital from the agents who own the capital goods. They maximize current profits of producing consumer goods given the prices of consumer goods, capital goods, rental on capital, and wage rate. Markets for labor and capital services are competitive. New investments are carried out directly by the two agents utilizing publicly available investment technology, so all intertemporal decisions are made by the agents while firms carry out current production.

2.1. The Technology

The model has two sectors. The production of consumer goods is carried out by competitive firms while the production of capital goods is carried out *directly by the agents* in their own facilities using only consumer goods as inputs. This simple nature of the capital goods sector enables us to use a rather simple notation, as follows: P_t is the price of consumption goods (the "price level") at t; $P_t/P_{t-1} = \pi_t = 1 +$ the rate of inflation at t; K_{t-1} is the real capital stock employed in the production of consumer goods at t; W_t^N is the nominal wage at t; $\tilde{q}_t^s$ is the nominal price at t of a unit of capital goods installed; $q_t^s = \tilde{q}_t^s/P_t$ is the real price of capital goods at t; L_t is the labor input in the production of consumer goods at t; N_t is the input at t, in units of consumption goods, into the production of capital goods; Y_t is the real output of consumer goods at t; and I_t is the real output of new capital goods produced by the investment technology.

2.1.1. OUTPUT AND PRODUCTIVITY

There are a large number of identical competitive firms, and aggregate output of consumer goods is defined by a standard production function:

$$Y_t = Av_t(K_{t-1})^\sigma (\xi_t L_t)^{1-\sigma} \quad \text{(typically with } A = 1\text{)}. \tag{1}$$

The productivity process $\{\xi_t, t = 1, 2, \ldots\}$ is a *deterministic* trend process satisfying

$$\xi_{t+1}/\xi_t = v^*, \tag{1a}$$

whereas random productivity $\{v_{t+1}, t = 1, 2, \ldots\}$ is a Markov process of the form

$$\log(v_{t+1}) = \lambda_v \log(v_t) + \rho_{t+1}^v, \qquad \rho_t^v \sim N\left(0, \sigma_v^2\right) \quad \text{iid.} \tag{1b}$$

Most productivity studies set the quarterly mean rate of technological change at $v^* = 1.0045$, and this is the value we use in all our computations.

The key parameters for the traditional RBC literature are (λ_v, σ_v) set at $\lambda_v = 0.976$, $\sigma_v = 0.0072$ for quarterly data. Empirical evidence suggests that σ_v is a fraction of 0.0072 and for low values of this parameter the RBC model fails to generate volatility (see King and Rebelo, 1999, fig. 8, p. 965). Accordingly, we set these parameters in our model at $\sigma = 0.35$, $\lambda_v = 0.976$, $\sigma_v = 0.002$.

The aggregate capital accumulation equation is defined by

$$K_t = (1 - \delta)K_{t-1} + I_t, \tag{2}$$

where δ is the rate of depreciation and I_t are new units of capital placed into production at $t + 1$ (inputs on these units would have been expanded at t) by the investment goods sector. Most studies set $\delta = 0.025$ and this is the value we use. Define $k_t = K_t/\xi_t$, $i_t = I_t/\xi_t$, $w_t = W_t^N/\xi_t P_t$, and $g_t = Y_t/\xi_t$; hence,

$$g_t = \frac{Y_t}{\xi_t} = A v_t L_t \left(\frac{k_{t-1}}{v^* L_t}\right)^\sigma \tag{3a}$$

and

$$k_t = (1 - \delta)\frac{k_{t-1}}{v^*} + i_t. \tag{3b}$$

Since $q_t^s = \bar{q}_t^s/P_t$ is the real price of a unit of capital, in equilibrium the real wage rate w_t and the rental rate on capital R_t are defined by the marginal productivity conditions in consumption goods:

$$w_t = A v_t (1 - \sigma)\left(\frac{k_{t-1}}{v^* L_t}\right)^\sigma, \tag{4a}$$

$$R_t = A v_t \sigma \left(\frac{k_{t-1}}{v^* L_t}\right)^{\sigma-1} - \delta q_t^s. \tag{4b}$$

2.1.2. THE INVESTMENT SECTOR

Heterogeneity of investment activities and uncertainty in the productivity of the investment goods sector are central features of our model. Investments embody new technologies, new products, and new processes of production. Such new capacities may or may not be successfully incorporated into the economy. These reflect the fact that most new enterprises fail and a significant number of investments in new

technologies do not achieve their goal. Uncertainty in the production of capital goods also incorporates the risk associated with the *diversification* strategy of firms across different product lines or economic sectors. In such cases the risk reflects many different factors. For example, drilling for oil is associated with the direct risk of finding oil. The risk of building a massive network of fiber optics communication is associated with the uncertainty of future demand for such capacities, as has been recently illustrated. Since our model does not have heterogeneous commodities to reflect all these risks, we introduce this uncertainty in the form of *the number of units of realized capital goods that result from an input of one unit of consumption goods.*

Optimal investment decisions depend upon expectations of returns among alternatives. Although we could have introduced a large number of investment activities, simplicity suggests that two activities suffice to exhibit all the essential features. The two constant-returns-to-scale projects transform consumer goods into a random number of units of capital goods depending upon the realized state φ_t. The random output of the two activities is specified in the following way:

TECHNOLOGY I (M):

Input: 1 unit of consumption good; $\text{Output}^M_{t+1} = \kappa^M_{t+1} = d_1 + \dfrac{d_2 - d_1}{1 + e^{-\chi^\nu \varphi_{t+1}}}$, (5a)

TECHNOLOGY II (S):

Input: 1 unit of consumption good; $\text{Output}^S_{t+1} = \kappa^S_{t+1} = s_1 + \dfrac{s_2 - s_1}{1 + e^{-\chi^\nu \varphi_{t+1}}}$, (5b)

where the empirical distribution of the shock is $\varphi_t \sim N(0, \sigma^2_\varphi)$ iid. The REE is defined to be the equilibrium under the belief that $\varphi_t \sim N(0, \sigma^2_\varphi)$ is the truth. In an RBE agents know that the long-term frequency of the states is represented by $\varphi_t \sim N(0, \sigma^2_\varphi)$ but they do not know the true process.

We assume that relative to the distribution $\varphi_t \sim N(0, \sigma^2_\varphi)$, it is optimal to fully diversify investments *leading to a perfect hedged position and no aggregate uncertainty.*[2] To clarify this issue consider the case $d_1 = 0.85$, $d_2 = 0.95$, $s_1 = 1.05$, $s_2 = 0.75$, $\chi^\varphi = 3$ used in most of the simulations in this chapter. If an agent invests 1 unit with proportions $I^M = \frac{3}{4}$, $I^S = \frac{1}{4}$ he ensures production of 0.90 units of capital in *all states.* His expected value is also 0.90, so diversification is optimal. When all agents believe that $\varphi_t \sim N(0, \sigma^2_\varphi)$ is the truth, they will have a fully diversified investment portfolio with *no fluctuations in the aggregate level or realized aggregate cost of investments.*

In an economy with diverse beliefs agents do not fully diversify. They believe that the mean value function of $\{\varphi_t, t = 1, 2, 3, \ldots\}$ varies over time (which is true), and they have subjective models about this value. Since the long-term moments of φ_t are $(0, \sigma^2_\varphi)$ with 0 autocorrelation, the rationality of belief conditions require

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2. Our assumption of the optimality of a perfect hedge is made for convenience only. It is motivated by computational ease since the perturbation method used here enables us to compute the equilibrium using the steady state as a reference point. The case where full diversification is not optimal raises only computational difficulties, not conceptual ones.

the agents' models to be statistically compatible with these facts. However, at each date they do not believe that the mean value is zero, so their optimal investment portfolio will vary depending upon these beliefs.

2.2. The Infinitely Lived Agents

We first introduce the following notation for $k = 1, 2$: C_t^k is the consumption of k at t; $\ell_t^k = 1 - L_t^k$ is the leisure of k at t; K_t^{kd} is the amount of capital stock purchases by k on the open market at t; K_{t-1}^k is the amount of capital owned by k at t and used in production at t; N_t^{kM} is the input (in units of consumption goods) of k in investments technology M at t; N_t^{kS} is the input (in units of consumption goods) of k in investments technology S at t; $N_t^k = N_t^{kM} + N_t^{kS}$ is the total input of agent k in investments technology at t; I_t^k is the total *output of* new investments (in *units of capital goods*) of k at t; B_t^k is the amount of a one-period nominal bill purchased by agent k at t; q_t^b is the price of a one-period bill at t, which is a discount price; M_t^k is the amount of money held by agent k at t; $M_t/M_{t-1} = \varrho_t$ is the random growth rate of money supply when it is a policy instrument; and H_t is the history of all observables up to t.

Before formulating the agents' optimization we need to clarify *the issue of timing*. At date t agent k invests (N_t^{kM}, N_t^{kS}) and at the start of date $t + 1$ the random realization takes place in the investment goods sector, *before* the rate of technical progress is realized. The new capital I_t^k joining production is added to $K_t^d = K_t^{1d} + K_t^{2d}$ to form the capital K_t used in production at date $t + 1$. This implies $I_t^k = \kappa_{t+1}^M N_t^{kM} + \kappa_{t+1}^S N_t^{kS}, k = 1, 2$, which is somewhat odd notation. This odd feature is unavoidable since we compress two random realizations into one date: first of φ_t and then of v_t, the rate of technological progress. Now, denote the inputs into the investment sector by $N_t = N_t^1 + N_t^2$. Output of new investments at date t is affected by φ_t and is defined by

$$I_{t-1}^k = \kappa_t^M N_{t-1}^{kM} + \kappa_t^S N_{t-1}^{kS}, \qquad k = 1, 2.$$

We thus define I_t^N, the *value* of new investments placed into production at date t, to be

$$I_t^N = \tilde{q}_t^s \left(I_{t-1}^1 + I_{t-1}^2 \right) \quad \left(\text{while noting that } I_{t-1} = I_{t-1}^1 + I_{t-1}^2 \right)$$

and Y_t^N, which is GNP, by $Y_t^N = Y_t - N_t + I_t^N$. Define $g^N = Y_t^N/\xi_t$, $n_t = N_t/\xi_t$, $i_t^N = I_t^N/\xi_t$, and the income identity becomes $g_t^N = g_t - n_t + i_t^N$.

Now, for any probability belief Q^k of agent k, his problem is to maximize the utility

$$\max E_{Q^k} \left\{ \sum_{t=1}^{\infty} \beta_k^{t-1} \frac{1}{1 - \gamma_k} \left\{ \left[C_t^k \left(\ell_t^k \right)^\varsigma \right]^{1-\gamma_k} \right. \right. \tag{6a}$$

$$\left. \left. + \left(M_t^k/P_t \right)^{1-\gamma_k} \right\} \Big| H_t \right\}, \qquad 0 < \beta_k < 1,$$

subject to the budget constraint

$$P_t C_t^k + K_t^{kd} \tilde{q}_t^s + \left(N_t^{kM} + N_t^{kS}\right) P_t + B_t^k q_t^b - \left(1 - \ell_t^k\right) W_t^N \qquad (6b)$$

$$- K_{t-1}^k \left(\tilde{q}_t^s + R_t P_t\right) - B_{t-1}^k + M_t^k - M_{t-1}^k \varrho_t = 0,$$

$$I_{t-1}^k = \kappa_t^M N_{t-1}^{kM} + \kappa_t^S N_{t-1}^{kS}, \quad N_t^{kM} \geq 0, \quad N_t^{kS} \geq 0, \qquad (6c)$$

$$K_{t-1}^k = K_{t-1}^{kd} + I_{t-1}^k. \qquad (6d)$$

Note the distinction between K_t^{kd} purchased at t on the open market and K_{t-1}^k used in production at t and owned by the agent at that time. Normalize the problem by $c_t^k = C_t^k/\xi_t$, $b_t^k = B_t^k/P_t\xi_t$, $M_t/P_t\xi_t = m_t$, $M_t^k/P_t\xi_t = m_t^k$, $M_{t-1}^k/P_t\xi_t = m_{t-1}^k/\pi_t v_t$, $n_t^k = N_t^k/\xi_t$, $i_t^k = I_t^k/\xi_t$, $k_t^k = K_t^k/\xi_t$, $K_t^{kd} = K_t^{kd}/\xi_t$. With the use of (5a) and (5b) and $P_{t+1}/P_t = \pi_{t+1}$ the maximization problem becomes

$$\max E_Q \sum_{t=1}^{\infty} \beta_k^{t-1} \frac{1}{1-\gamma_k} \left\{ \left[c_t^k \xi_t \left(\ell_t^k\right)^\varsigma\right]^{1-\gamma_k} + \left(m_t^k \xi_t\right)^{1-\gamma_k} \right\}, \quad 0 < \beta_k < 1, \quad (6a')$$

subject to

$$c_t^k = \left(1 - \ell_t^k\right) w_t + \frac{k_{t-1}^k}{v^*}(q_t^s + R_t) + \frac{m_{t-1}^k \varrho_t + b_{t-1}^k}{v^* \pi_t} - k_t^{kd} q_t^s \qquad (6b')$$

$$- \left(n_t^{kM} + n_t^{kS}\right) - b_t^k q_t^b - m_t^k,$$

$$i_{t-1}^k = \kappa_t^M n_{t-1}^{kM} + \kappa_t^S n_{t-1}^{kS}, \quad n_{t-1}^{kM} \geq 0, \quad n_{t-1}^{kS} \geq 0, \qquad (6c')$$

$$k_{t-1}^k = k_{t-1}^{kd} + i_{t-1}^k. \qquad (6d')$$

To simplify the Euler equations we ignore the inequality constraints in (6c') and handle them only as computational issues raised by the model.[3] Hence, the first-order conditions for labor supply are

$$c_t^k = (1/\varsigma)\ell_t^k w_t. \qquad (7a)$$

Next, the first-order condition with respect to capital purchased on the open market k_t^{kd} is

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3. An Appendix B on the computational model is not included in this published version. Any interested reader can obtain this appendix by downloading the prepublished version of this chapter, which is posted on the homepage of the first author at http://www.stanford.edu/~mordecai. The prepublished version also contains Appendix A, which provides a short account of the theory of rational beliefs and other technical details not included here.

$$(c_t^k)^{-\gamma_k} \left[(\ell_t^k)^\varsigma\right]^{1-\gamma_k} q_t^s = \beta_k E_{Q^k} (c_{t+1}^k)^{-\gamma_k} \left[(\ell_{t+1}^k)^\varsigma\right]^{1-\gamma_k} \left[\frac{q_{t+1}^S + R_{t+1}}{(v^*)^{\gamma_k}}\right]. \quad (7b)$$

The optimality condition with respect to n_t^{kM} is

$$(c_t^k)^{-\gamma_k} \left[(\ell_t^k)^\varsigma\right]^{1-\gamma_k} = \beta_k E_{Q^k} (c_{t+1}^k)^{-\gamma_k} \left[(\ell_{t+1}^k)^\varsigma\right]^{1-\gamma_k} \left[\frac{q_{t+1}^S + R_{t+1}}{(v^*)^{\gamma_k}} \kappa_{t+1}^M\right], \quad (7c)$$

and the condition with respect to n_t^{kS} is

$$(c_t^k)^{-\gamma_k} \left[(\ell_t^k)^\varsigma\right]^{1-\gamma_k} = \beta_k E_{Q^k} (c_{t+1}^k)^{-\gamma_k} \left[(\ell_{t+1}^k)^\varsigma\right]^{1-\gamma_k} \left[\frac{q_{t+1}^S + R_{t+1}}{(v^*)^{\gamma_k}} \kappa_{t+1}^S\right]. \quad (7d)$$

The first-order condition with respect to b_t^k is

$$(c_t^k)^{-\gamma_k} (\ell_t^k)^{\varsigma(1-\gamma_k)} q_t^b = \beta_k E_{Q^k} (c_{t+1}^k)^{-\gamma_k} (\ell_{t+1}^k)^{\varsigma(1-\gamma_k)} \frac{1}{(v^*)^{\gamma_k} \pi_{t+1}}. \quad (7e)$$

Finally, the optimum with respect to money holdings requires

$$(c_t^k)^{-\gamma_k} (\ell_t^k)^{\varsigma(1-\gamma_k)} - (m_t^k)^{-\gamma_k} = \beta_k E_{Q^k} \left[(c_{t+1}^k)^{-\gamma_k} (\ell_{t+1}^k)^{\varsigma(1-\gamma_k)}\right] \frac{\varrho_{t+1}}{(v^*)^{\gamma_k} \pi_{t+1}}. \quad (7f)$$

2.3. Monetary Policy

The monetary "policy" in the model is the familiar monetary injection: The central bank increases the money supply by a random amount $(\varrho_{t+1} - 1)M_t$. Hence, M_t^k, which is the date t money holding of agent k, increases to $\varrho_{t+1}M_t^k$ between date t and date $t + 1$. Agents observe the monetary shock and since they observe the real shocks (v_t, φ_t), the exogenous state is fully observed.

Random variations in the money supply do not constitute a serious monetary policy that may be pursued by any central bank. This chapter is a theoretical investigation in which an exogenous money supply is a simple device for studying the effect of beliefs on economic fluctuations. This approach will also enable us to discuss the role that a real monetary policy should play in economic stabilization.

2.4. Equilibrium

For each set of probability beliefs (Q^1, Q^2) of the agents on infinite sequences of observed variables, a monetary equilibrium of the economy is defined by equations: (3a), (3b), (4a), (4b), (6b′), and (7a)–(7f), as well as by the following additional conditions:

Money growth: The money supply satisfies

$$m_t = (\varrho_t / \pi_t v_t)\, m_{t-1}.$$

Market clearing conditions: Given the accounting in (6a)–(6d) we aggregate to establish two equations, one accounting identity, specifying the amount of capital employed at date t:

$$K_{t-1} = K_{t-1}^1 + K_{t-1}^2,$$

and a market clearing condition in the market for installed capital at date t:

$$K_t^{1d} + K_t^{2d} = K_{t-1}(1 - \delta).$$

The dynamics of capital employed is then defined by

$$K_t = K_{t-1}(1 - \delta) + I_t,$$

$$I_{t-1} = I_{t-1}^1 + I_{t-1}^2 = \kappa_t^M \left(N_{t-1}^{1M} + N_{t-1}^{2M} \right) + \kappa_t^S \left(N_{t-1}^{1S} + N_{t-1}^{2S} \right).$$

After normalization we thus have the identities

$$k_t^1 + k_t^2 = k_t \qquad \text{for all } t,$$

$$i_{t-1}^k = \kappa_t^M n_{t-1}^{kM} + \kappa_t^S n_{t-1}^{kS}, \qquad k = 1, 2 \qquad \text{for all } t,$$

and the market clearing conditions

$$k_t = k_{t-1} \left[(1 - \delta)/v^* \right] + i_t \qquad \text{for all } t, \tag{8a}$$

$$k_t^{1d} + k_t^{2d} = k_{t-1} \left[(1 - \delta)/v^* \right] \qquad \text{for all } t, \tag{8b}$$

$$b_t^1 + b_t^2 = 0 \qquad \text{for all } t, \tag{8c}$$

$$\left(1 - \ell_t^1 \right) + \left(1 - \ell_t^2 \right) = L_t \qquad \text{for all } t, \tag{8d}$$

$$m_t^1 + m_t^2 = m_t \qquad \text{for all } t. \tag{8e}$$

We turn now to the central question of the beliefs of the agents.

3. A RATIONAL BELIEF EQUILIBRIUM

We now construct the RBE and explain the family of rational beliefs that we study. For a detailed account of the method of constructing an RBE see Kurz and Motolese (2001). In Section 3.3 we explain the method of assessment variables, used extensively in this chapter, for describing a rational belief.

3.1. The Equilibrium Map

Our procedure is to construct an RBE for the economy, use perturbation methods to compute it, and study its dynamic properties via simulations. However, to define an RBE we have to specify the beliefs of the agents and this cannot be done without saying something about the empirical distribution implied by that RBE and its induced stationary measure.[4] To break this circularity we start by studying the *structure* of an RBE and use it for a general specification of the *structure* of the stationary measure around which we construct the beliefs of the agents (for details of this approach, see Kurz and Motolese, 2001, sect. 2.4). Such a constructive procedure is possible only when we study a specific family of rational beliefs, which is the case in this chapter. We carry out this procedure in several steps, starting with the equilibrium map.

Recall that in the optimization (6a′)–(6b′) agent k derives optimal decisions by using belief Q_y^k conditional on public information and on y_t^k, the value of his own assessment variable. *We assume that all portfolios and exogenous shocks* $(v_t, \varrho_t, \varphi_t)$ *are observable*. The observables are $x_t = (v_t, \varrho_t, \varphi_t, k_{t-1}^{1d}, n_{t-1}^{1M}, n_{t-1}^{1S}, b_{t-1}^1, k_{t-1}^{2d}, n_{t-1}^{2M}, n_{t-1}^{2S}, b_{t-1}^2, m_{t-1}^1, m_{t-1}^2, q_t^s, q_t^b, \pi_t)$. To simplify we denote lagged endogenous variables $x_{t-1}^E = (k_{t-1}^{1d}, n_{t-1}^{1M}, n_{t-1}^{1S}, b_{t-1}^1, m_{t-1}^1, k_{t-1}^{2d}, n_{t-1}^{2M}, n_{t-1}^{2S}, b_{t-1}^2, m_{t-1}^2)$. Agent k will thus condition on (x_t, y_t^k) and hence optimal decisions are functions of the form

$$k_t^{kd} = k^{kd}\left(v_t, \varrho_t, \varphi_t, x_{t-1}^E, q_t^s, q_t^b, \pi_t, y_t^k\right) \qquad k = 1, 2, \qquad (9a)$$

$$n_t^{kM} = i^{kM}\left(v_t, \varrho_t, \varphi_t, x_{t-1}^E, q_t^s, q_t^b, \pi_t, y_t^k\right) \qquad k = 1, 2, \qquad (9b)$$

$$n_t^{kS} = i^{kS}\left(v_t, \varrho_t, \varphi_t, x_{t-1}^E, q_t^s, q_t^b, \pi_t, y_t^k\right) \qquad k = 1, 2, \qquad (9c)$$

$$b_t^k = b^k\left(v_t, \varrho_t, \varphi_t, x_{t-1}^E, q_t^s, q_t^b, \pi_t, y_t^k\right) \qquad k = 1, 2, \qquad (9d)$$

$$m_t^k = m^k\left(v_t, \varrho_t, \varphi_t, x_{t-1}^E, q_t^s, q_t^b, \pi_t, y_t^k\right) \qquad k = 1, 2, \qquad (9e)$$

$$\ell_t^k = \ell^k\left(v_t, \varrho_t, \varphi_t, x_{t-1}^E, q_t^s, q_t^b, \pi_t, y_t^k\right) \qquad k = 1, 2. \qquad (9f)$$

Market clearing conditions imply that the equilibrium price process $\{(q_t^s, q_t^b, \pi_t), t = 1, 2, \ldots\}$ is thus defined by a map of the form

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4. The empirical distribution of the observable variables or their moments induce a probability measure over infinite sequences of observables that is central to the theory of rational beliefs. A general definition and construction of this probability measure is explained in Kurz (1997a) or in Appendix A of the prepublished version available at the web address provided in footnote 3. Any statement in the text about "the stationary measure" or "the empirical distribution" is always a reference to this probability measure. Its centrality to the theory arises from the fact that this probability is derived from public information and hence *the stationary measure is known to all agents and agreed upon by all to reflect the empirical distribution of equilibrium quantities.*

$$\begin{pmatrix} q_t^s \\ q_t^b \\ \pi_t \end{pmatrix} = \Phi\left(v_t, \varrho_t, \varphi_t, y_t^1, y_t^2, x_{t-1}^E\right). \tag{10}$$

Equation (10) reveals that the volatility of equilibrium prices is determined by three factors: exogenous states $(v_t, \varrho_t, \varphi_t)$, lagged endogenous variables x_{t-1}^E, and the states of belief (y_t^1, y_t^2). "Endogenous uncertainty" was defined by Kurz (1974, 1997a) as that component of volatility that is generated by the distribution of beliefs in the market, represented by (y_t^1, y_t^2). In Section 3.3 we explain that an assessment variable y_t^k is a simple mathematical representation of a private state of belief. The term y_t^k is a privately perceived parameter of the agent's belief, *uniquely* defining his conditional probability belief over observables. The states of an agent's belief are restricted by the rationality of belief conditions, and the restrictions applicable to our model are specified later.

The Euler equations and the definition of consumption in (6b′)–(6d′) show that at any date t agent k has to forecast three categories of variables discussed later: (1) exogenous variables $(v_{t+1}, \varrho_{t+1}, \varphi_{t+1})$ conditional upon information at t and his assessment y_t^k; (2) his own decisions at $t+1$, $(k_t^{kd}, n_{t+1}^{kM}, n_{t+1}^{kS}, b_{t+1}^k, \ell_{t+1}^k, m_{t+1}^k)$, based on his optimal decision functions and a forecast of his assessment y_{t+1}^k; (3) other endogenous variables, particularly prices $(q_{t+1}^s, q_{t+1}^b, \pi_{t+1})$, conditional upon information at t and his assessment y_t^k. The rationality of belief conditions require that all moments of an agent's subjective model be exactly the same as the moments of the stationary measure derived from the empirical distribution of the observables. Hence, a formulation of the beliefs (or perception models) of agents necessitates our specifying first the general structure of the stationary measure.

3.2. Construction of the Stationary Measure

Our RBE is an equilibrium of a stochastic economy that fluctuates around steady-state values denoted by an asterisk. Since equilibrium quantities satisfy the map (10), the empirical distribution is determined *by the equilibrium map and the long-term behavior of* $(v_t, \varrho_t, \varphi_t, y_t^1, y_t^2)$. Our method of constructing an RBE is to specify first the empirical distribution of $(v_t, \varrho_t, \varphi_t, y_t^1, y_t^2)$. Once this is done, we specify the beliefs of agents and impose the rationality conditions.

The true processes may exhibit nonstationary dynamics. We thus *assume* instead that the stationary measure of exogenous variables $(v_t, \varrho_t, \varphi_t)$ and of states of belief (y_t^1, y_t^2) have the following structure:

$$\log v_{t+1} = \lambda_v \log v_t + \rho_{t+1}^v, \qquad \rho_t^v \sim N(0, \sigma_v^2) \text{ iid}, \tag{11a}$$

$$\log \varrho_{t+1} = \log \varrho^* + \lambda_\varrho \left(\log \varrho_t - \log \varrho^*\right) + \rho_{t+1}^\varrho, \qquad \rho_t^\varrho \sim N\left(0, \sigma_\varrho^2\right) \text{ iid}, \tag{11b}$$

$$\varphi_{t+1} = \rho_{t+1}^\varphi, \qquad \rho_t^\varphi \sim N(0, \sigma_\varphi^2) \text{ iid}. \tag{11c}$$

The sequence of states $\{(y_t^1, y_t^2), t = 1, 2, \ldots\}$ is a *realization* of a stochastic process of the form

$$
\begin{pmatrix} y_t^1 \\ y_t^2 \end{pmatrix} = \begin{pmatrix} \rho_t^{y^1} \\ \rho_t^{y^2} \end{pmatrix}, \quad \begin{pmatrix} \rho_t^{y^1} \\ \rho_t^{y^2} \end{pmatrix} \sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{y^1}^2, & \sigma_{yy} \\ \sigma_{yy}, & \sigma_{y^2}^2 \end{pmatrix} \right] \text{ iid.} \quad (11d)
$$

Equations (11b)–(11d) are *hypothetical*. The assumptions that the investment sector shocks φ_t and the states of belief (y_t^1, y_t^2) *exhibit no long-run persistence* are made for computational simplicity. These assumptions are very strong and their implications will be evaluated later.

We make the following *additional simplifying assumptions:*

A1. $(\rho_t^v, \rho_t^\varrho, \rho_t^\varphi, \rho_t^{y^1}, \rho_t^{y^2})$ are mutually independent.
A2. $\varrho^* = v^*$; hence (v^*, v^*) are the steady-state values of (v_t, ϱ_t) in the riskless economy and the long-term average inflation rate is zero in both REE and RBE.
A3. As discussed earlier $\lambda_v = 0.976$, $\sigma_v = 0.002$.
A4. As monetary shocks and the stochastic investment technology are hypothetical, we specify $\sigma_\varphi = 1$, $\lambda_\varrho = 0.95$, $\sigma_\varrho = 0.0052$. This implies that the standard deviation of monetary shocks over time is 1 percent per quarter.

These assumptions are intended to enable a quantitative evaluation of the workings of our model by postulating persistence in the monetary shocks with a reasonable standard deviation. As for (y_t^1, y_t^2), we assume $\sigma_{y^1} = \sigma_{y^2} = 0.8$ and a correlation coefficient $\rho(y^1, y^2) = 0.95$, hence $\sigma_{yy} = 0.608$.

3.3. The Belief Structure

3.3.1. GENERAL ASSUMPTIONS

a. Properties of Assessment Variables
Assessment variables y_t^k for $k = 1, 2$ are the tools that we use to describe an agent's belief. We briefly review their properties here.

i. Definition. Assessment variables are artificial variables used to describe nonstationarity but without an intrinsic meaning of their own. Thus let X be a space of observables and suppose that we want to describe the nonstationarity of a dynamical system on the space of infinite sequences of observables $[(X)^\infty, \mathscr{B}((X)^\infty)]$, where $\mathscr{B}((X)^\infty)$ is the Borel σ-field of $(X)^\infty$. The conditional stability theorem (see Kurz and Schneider, 1996, and Nielsen, 1996) describes nonstationarity via artificial variables $y_t^k \in Y^k$ with a marginal probability space $[(Y^k)^\infty, \mathscr{B}((Y^k)^\infty), \mu]$. It postulates $(X \times Y^k)$ to be the state space, introduces a universal probability measure Q^k and space $[(X \times Y^k)^\infty, \mathscr{B}((X \times Y^k)^\infty), Q^k]$, and defines the desired nonstationary probability to be $Q_{y^k}^k \equiv Q^k[(\cdot) \mid y^k]$, the conditional probability of Q^k with respect to the sequence y^k. The term Q^k must satisfy the condition that for all $A \in \mathscr{B}((X)^\infty)$ and $B \in \mathscr{B}((Y)^\infty)$,

$$Q^k(A \times B) = \int_B Q^k_{y^k}(A)\mu(dy).$$

The *effective* conditional probability space $[(X)^\infty, \mathcal{B}((X)^\infty), Q^k_{y^k_t}]$ implies non-stationary dynamics of the observables since probabilities of events in $\mathcal{B}((X)^\infty)$ are not time independent: They change with the parameters y^k_t, which are time dependent. The advantage of using assessment variables is statistical since the stochastic structure above pins down the empirical regularity of y^k_t in relation to observables, and this regularity will be seen to be the basis for the rationality conditions. This approach is common in econometrics, where Y is the set of possible "regimes," y_t identifies the regime at t, and hence the y_t are viewed as "regime variables."

Although the space $[(X \times Y^k)^\infty, \mathcal{B}((X \times Y^k)^\infty), Q^k]$ is defined over a fixed set Y^k, this need not be the case. We could replace $(Y^k)^\infty$ by an infinite number of *different* spaces Y^k_j and define Q over the product space $[(X)^\infty \times \Pi^\infty_{j=1}Y^k_j]$. We can then define the desired nonstationary probability to be $Q^k_{y^k} \equiv Q^k[(\cdot) \mid y^k]$, which is conditional upon a sequence of *different* objects in $\Pi^\infty_{j=1}Y^k_j$. In short, assessment variables of an agent are privately perceived parameters that *the agent himself generates* for a description of his belief. They may be in different spaces at different times and in different spaces across agents so that there is no sense in which they can be compared. The quantity y^j_t is privately perceived by agent j and has meaning only to him; agent $k \neq j$ would not know what y^j_t means even if he could "observe" it.

ii. Economic Interpretation. $Q^k_{y_t}$ is the date t probability belief of future *observables* and y^k_t is used to describe how the agent's forecasts deviate from the stationary forecasts owing to his belief in nonstationarity. There are several issues to note. First, deviation from the stationary forecast is a judgment of "structural breaks" based on limited *recent* data. Such judgments are right or wrong, so in an RBE *rational agents are often wrong*. Second, in a nonstationary environment it is usually not possible to demonstrate with high likelihood that a belief was right, even in retrospect. Third, even if sufficient data become available to determine whether a change of structure had occurred, such information would arrive too late since all important decisions would have already been made.

iii. Statistical Simplicity. Rational agents do not deviate *systematically* from the stationary forecasts and to establish rationality of belief we need a statistical measure of an agent's deviation from these forecasts. Assessment variables provide a simple tool to measure such regularity.

iv. The Infinitely Lived Agent Is a Sequence of Decision-Makers, So Assessment Variables Are Not Subject to Rationality Conditions. An economy with changing technology and products is one in which the commodity space changes, and thus y^k_t at different times describes beliefs about different commodities or technologies. Hence, $y^k_{t_1}$ and $y^k_{t_2}$ for t_1 and t_2 in different time intervals are simply

different *objects* making comparisons impossible, so that rationality conditions cannot apply to y_t^k. The problem is that a model with an infinite dimensional commodity space and infinitely varying regimes is analytically untractable. An idealization that makes sense is to assume that the commodity space is fixed[5] and that y_t^k, a private parameter meaningful only to agent k, is *required* to have a consistent meaning through time. But then how do we model the fact that in reality an agent cannot "test" his theory and no rationality conditions should apply to the sequence of y_t^k? The solution is to consider the infinitely lived agent as consisting of *an infinite sequence of members (of a family or organization) each making economic decisions over a relatively short time*. Every decision-maker knows only his own y_t^k, but not those of his predecessors. Hence, even if an agent has data to convince himself that his theory is right or wrong, such data arrive too late to be useful since all important decisions have already been made.

b. Other Assumptions

A5. *Anonymity assumption*: In an economy with two agents the belief of each has an impact on prices. Agents act competitively and ignore the effect of their own beliefs on equilibrium prices.

Agents know that beliefs impact prices and their forecasting models have to forecast the impact of future distribution of beliefs on prices. To explain how this works in our model note that knowing the stationary measure means knowing all long-term conditional distributions given the observed variables. However, given fundamentals $(x_{t-1}^E, v_t, \varrho_t, \varphi_t)$, the variability of endogenous variables is *determined by the variability of the states of belief* (y_t^1, y_t^2). Since states of belief are not observed, their impact is deduced from the long-term distribution of endogenous variables conditional upon $(x_{t-1}^E, v_t, \varrho_t, \varphi_t)$. States of belief account for the higher volatility of the endogenous variables unexplained by $(x_{t-1}^E, v_t, \varrho_t, \varphi_t)$. We have named this component of volatility Endogenous uncertainty and this argument shows that in learning the stationary measure *agents also discover the component of the stationary measure induced by the distribution of beliefs*. For simplicity of computations we make the following assumption:

A6. *Forecasting assumption I*: Agents believe that the endogenous impact of market states of belief on prices and other endogenous variables is the same as the component that is estimated from the stationary measure.

We turn finally to our last simplifying assumption regarding the forecasting of prices in the model.

A7. *Forecasting assumption II*: Agents forecast prices using the map (10) and perceive the impact of states of belief on prices and other endogenous variables via a variable z_t with the same distributional properties as the long-run properties of the unobserved states of belief (y_t^1, y_t^2).

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5. Hence, one defines commodities by their attributes, such as "transportation equipment" instead of "airplanes" or "communication services" instead of "telephone." Over time, we then experience change in the cost of producing these services and their prices rather than change in the definitions of the products or services themselves.

Assumption A.7 is not entirely compatible with postulates of the RBE theory, which hold that agents do not know equilibrium maps or true probabilities; we make it in order to facilitate the computations of RBE. It amounts to assuming that our RBE are incomplete Radner equilibria (see Radner, 1972) with an expanded state space of unobserved states of beliefs. Markets are incomplete since agents have only two assets to trade market uncertainty.[6]

3.3.2. The Perception Models

Given a stationary measure, true probabilities play absolutely no role in defining equilibrium. What matters is what agents *know* and *how they perceive the future.* We have already selected the basic long-term statistics that agents know, and we now turn to their perceptions. Our strategy is to *specify the agents' beliefs and then rationalize them.* We start with the states of belief. Equation (11d) describes the long-term behavior of the *joint* states of belief but the rationality conditions require marginal distributions of y_t^k to have compatible asymptotic properties. Hence, we postulate that the state of belief of agent k, *on his own,* takes the form

$$y_t^k = \sigma_{z^k} z_t^{y^k}, \tag{12}$$

where $z_t^{y^k}$ is a realization of a normal iid random variable $\tilde{z}_t^{y^k} \sim N(0, 1)$. In the simulations we set the parameter $\sigma_{z^k} = 0.8$ for $k = 1, 2$.

a. Perceived Productivity Shocks and Their Rationality Conditions
The beliefs of agent k is expressed as a stochastic difference equation of the form

$$\log v_{t+1}^k = \lambda_v \log v_t^k + \lambda^{kv}\left(\varrho_t^k\right) y_t^k + \sigma_{z^{kv}}\left(\varrho_t^k\right)\hat{z}_{t+1}^{kv}, \qquad \hat{z}_t^{kv} \sim N(0, 1). \tag{13}$$

The agent believes that λ_v is as in (11a) but the mean value varies with y_t^k. Since the empirical distribution in (11a) exhibits no long-term covariance between productivity shocks, monetary shocks, and states of belief, rationality of belief must conform to that. Hence, rationality requires the statistics of observables generated by (13) and (11a) to be the same when the states of belief y_t^k are treated as unobserved shifts in the mean value function. No rationality conditions are imposed on y_t^k or on the correlation between y_t^k and $\log v_{t+1}^k$.

In (13) agents form beliefs that $\lambda^{kv}(\varrho_t^k)$ and $\sigma_{z^{kv}}(\varrho_t^k)$ depend upon ϱ_t^k. We write ϱ_t^k but rationality requires that for observables, *perception equals realization* so $\varrho_t^k = \varrho_t$. Given this, the basic rationality condition can be stated simply:

The empirical distribution of $\quad \lambda^{kv}(\varrho_t)y_t^k + \sigma_{z^{kv}}(\varrho_t)\hat{z}_{t+1}^{kv}\quad$ is $\quad N(0, \sigma_v^2)$. (14)

We use a specific functional form in which monetary shocks impact $\lambda^{kv}(\varrho_t)$ and this function is

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6. For a discussion of the relationship between an RBE and other equilibrium concepts, including sunspot equilibria, see Kurz and Motolese (2001, sect. 2.2d, pp. 513–7).

$$\lambda^{kv}(\varrho_t) = \frac{\lambda^{kv}}{1 + e^{-\chi^\varrho(\log \varrho_1)}}. \qquad (14a)$$

In (14a) the function is increasing in the size of monetary shocks and hence such shocks increase the effect of y_t^k on the perceived mean value of $\log v_{t+1}^k$ in (13). Given our assumptions in this chapter, we show that this belief is rationalized if we require that conditional upon ϱ_t^k, the second empirical moment of $\lambda^{kv}(\varrho_t)y_t^k + \sigma_{z^{kv}}(\varrho_t)\hat{z}_{t+1}^{kv}$ equals σ_v^2 or

$$\left[\lambda^{kv}\left(\varrho_t^k\right)\right]^2 \sigma_{z^k}^2 + \left[\sigma_{z^{kv}}\left(\varrho_t^k\right)\right]^2 = \sigma_v^2. \qquad (14b)$$

Condition (14b) pins down the functional form of $\sigma_{z^{kv}}(\varrho_t^k)$.

LEMMA: Under assumptions A.1–A.4, condition (14b) rationalizes the belief (13) with (14a) and hence the empirical distribution of $\lambda^{kv}(\varrho_t)y_t^k + \sigma_{z^{kv}}(\varrho_t)\hat{z}_{t+1}^{kv}$ is $N(0, \sigma_v^2)$.

PROOF: By (12)–(13) we have that y_t^k is a realization of a $N(0, \sigma_{z^k}^2)$ and $\hat{z}_t^{kv} \sim N(0, 1)$; hence the time averages of y_t^k and of $\hat{z}_t^{kv}$ are both 0. Let $\Upsilon_t^{kv} = \lambda^{kv}(\varrho_t)y_t^k + \sigma_{z^{kv}}(\varrho_t)\hat{z}_{t+1}^{kv}$ and observe that given ϱ_t the empirical distribution of Υ_t^{kv} is normal since it is a linear combination of two independent normal variables. We claim that this distribution is $N(0, \sigma_v^2)$. That is, if we denote the stationary measure by P_m then our claim is that $P_m(\Upsilon_t^{kv} \mid \varrho_t) = N(0, \sigma_v^2)$ *independent of* ϱ_t! To see why this is so note first that

$$E_{P_m}\left(\Upsilon_t^{kv} \mid \varrho_t\right) = \lambda^{kv}(\varrho_t)\left(\lim_{n \to \infty}\frac{1}{n}\sum_{t=1}^{n}y_t^k\right) + \sigma_{z^{kv}}(\varrho_t)\left(\lim_{n \to \infty}\frac{1}{n}\sum_{t=1}^{n}\hat{z}_{t+1}^{kv}\right) = 0.$$

Then by (14b)

$$E_{P_m}\left[(\Upsilon_t^{kv})^2 \mid \varrho_t\right] = \left[\lambda^{kv}(\varrho_t)\right]^2\left[\lim_{n \to \infty}\frac{1}{n}\sum_{t=1}^{n}(y_t^k)^2\right]$$

$$+ \sigma_{z^{kv}}^2(\varrho_t)\left[\lim_{n \to \infty}\frac{1}{n}\sum_{t=1}^{n}(\hat{z}_{t+1}^{kv})^2\right] = \sigma_v^2,$$

and since a normal distribution is characterized by these two moments the claim is proved. But now observe that if we denote the unconditional density of ϱ by $P_m(\varrho)$ we can conclude that

$$P_m\left(\Upsilon_t^{kv}\right) = \int P_m\left(\Upsilon_t^{kv} \mid \varrho_t\right) P_m(\varrho_t)d(\varrho_t)$$

$$= \int \left[N\left(0, \sigma_v^2\right)\right] P_m(\varrho_t)d(\varrho_t) = N\left(0, \sigma_v^2\right).$$

It is useful to specify four implications of the rationality conditions that have been established:

Mean:
The time average of $\quad \lambda^{kv}\left(\varrho_t^k\right) y_t^k + \sigma_{z^{kv}}\left(\varrho_t^k\right) \hat{z}_{t+1}^{kv} \quad$ is 0. $\qquad$ (15a)

Variance:
The time average of $\quad \left(\lambda^{kv}\left(\varrho_t^k\right) y_t^k + \sigma_{z^{kv}}\left(\varrho_t^k\right) \hat{z}_{t+1}^{kv}\right)^2 \quad$ is σ_v^2. $\qquad$ (15b)

Serial correlation:
The autocorrelation of $\quad \lambda^{kv}\left(\varrho_t^k\right) y_t^k + \sigma_{z^{kv}}\left(\varrho_t^k\right) \hat{z}_{t+1}^{kv} \quad$ is 0. $\qquad$ (15c)

Covariance:
The covariance between $\quad \lambda^{kv}\left(\varrho_t^k\right) y_t^k + \sigma_{z^{kv}}\left(\varrho_t^k\right) \hat{z}_{t+1}^{kv} \quad$ and ϱ_t is 0. $\qquad$ (15d)

In the simulations we set the values of $(\lambda^{kv}, \chi^\varrho)$ and (14b) determines the functional form of $\sigma_{z^{kv}}(\varrho_t^k)$. Earlier we explained the choice of $\lambda_v = 0.976$, $\sigma_v = 0.002$ and $\sigma_{z^k} = 0.8$ for $k = 1, 2$. Hence, the feasible range for λ^{kv} is very small. In the simulations we set $\lambda^{kv} = 0.002$ and $\chi^\varrho = 20$ (since $\log \varrho_t^k$ fluctuates mostly between 0.03 and -0.03). This effect is so small that for all practical purposes the reader may assume $\lambda^{kv} = 0$.

b. Perceived Monetary Shocks and Their Rationality Conditions

Analogous to (13), the beliefs of agents about future monetary shocks are expressed by

$$\log \varrho_{t+1}^k = \log v^* + \lambda_\varrho \left(\log \varrho_t^k - \log v^*\right) + \lambda^{k\varrho}\left(\varrho_t^k\right) y_t^k$$
$$+ \sigma_{z^{k\varrho}}\left(\varrho_t^k\right) \hat{z}_{t+1}^{k\varrho}, \quad \hat{z}_t^{k\varrho} \sim N(0, 1). \tag{16}$$

To rationalize this belief we proceed as in (14)–(15) above. The rationality condition is again

The empirical distribution of $\quad \lambda^{k\varrho}\left(\varrho_t^k\right) y_t^k + \sigma_{z^{k\varrho}}\left(\varrho_t^k\right) \hat{z}_{t+1}^{k\varrho} \quad$ is $N\left(0, \sigma_\varrho^2\right)$. (16a)

With $\varrho_t^k = \varrho_t$ the functional form for the effects of monetary shocks on $\lambda^{k\varrho}(\varrho_t)$ is

$$\lambda^{k\varrho}\left(\varrho_t\right) = \frac{\lambda^{k\varrho}}{1 + e^{-\chi^\varrho(\log \varrho_t)}}. \tag{16b}$$

In (16b) the function is increasing in the size of monetary shocks and hence such shocks increase the effect of y_t^k on the perceived mean value of $\log \varrho_{t+1}^k$ in (16). Using an argument as in the lemma above, this belief can be rationalized by the condition

$$\left[\lambda^{k\varrho}(\varrho_t)\right]^2 \sigma_{z^k}^2 + \left[\sigma_{z^{k\varrho}}(\varrho_t)\right]^2 = \sigma_\varrho^2. \tag{16c}$$

We have four analogous implications of rationality:

Mean:

The time average of $\quad \lambda^{k\varrho}(\varrho_t)y_t^k + \sigma_{z^{k\varrho}}(\varrho_t)\hat{z}_{t+1}^{k\varrho}\quad$ is 0.   (16d)

Variance:

The time average of $\quad \left(\lambda^{k\varrho}(\varrho_t)y_t^k + \sigma_{z^{k\varrho}}(\varrho_t)\hat{z}_{t+1}^{k\varrho}\right)^2\quad$ is σ_v^2.   (16e)

Serial correlation:

The autocorrelation of $\quad \lambda^{k\varrho}(\varrho_t)y_t^k + \sigma_{z^{k\varrho}}(\varrho_t)\hat{z}_{t+1}^{k\varrho}\quad$ is 0.   (16f)

Covariance:

The covariance between $\quad \lambda^{k\varrho}(\varrho_t)y_t^k + \sigma_{z^{k\varrho}}(\varrho_t)\hat{z}_{t+1}^{k\varrho}\quad$ and ϱ_t is 0.   (16g)

Equations (16b) and (16c) restrict the value of $\lambda^{k\varrho}$ and $\sigma_{z^{k\varrho}}(\varrho_t^k)$ that the agent can select in (16). We have already selected $\lambda_\varrho = 0.95$, $\sigma_\varrho = 0.0052$, $\sigma_{z^k} = 0.8$, $\chi^\varrho = 20$ and we now add $\lambda^{k\varrho} = 0.01$. Again, this effect is very small and may be disregarded.

c. Perceived Investment Sector Shocks and Their Rationality Conditions

Perceived shocks in the investment sector *are the main propagation mechanism of the model*. If agents believe that the true φ_t process is iid as in (11c), they fully diversify portfolios with $n_t^{kS}/(n_t^{kS}+n_t^{kD}) = \frac{1}{4}$ resulting in a *perfect hedged position* and investment of $0.9(n_t^{kS} + n_t^{kD})$ *independent* of φ_t. In this REE fluctuations in φ_t have no effect. Hence, the reference steady state is a risky economy in which portfolios are perfectly hedged. In an RBE agents do not believe that (11c) is the truth and hence fluctuations occur: When $y_t^k > 0$ the distribution of k's belief moves *in favor of M* and when $y_t^k < 0$ the distribution moves *in favor of S*. We postulate that the agents perceive shifts in the mean value of φ_t in accordance with y_t^k and their beliefs take the form

$$\varphi_{t+1}^k = \lambda^{k\varphi}\left(\varrho_t^k\right)y_t^k + \sigma_{z^{k\varphi}}\left(\varrho_t^k\right)\hat{z}_{t+1}^{k\varphi}, \quad \hat{z}_t^{k\varphi} \sim N(0,1). \tag{17}$$

Note that variations in the values of y_t^k impact the perceived mean of φ_t^k not of $\log(\varphi_t^k)$. As before, the basic rationality condition is

The empirical distribution of $\quad \lambda^{k\varphi}\left(\varrho_t^k\right)y_t^k + \sigma_{z^{k\varphi}}\left(\varrho_t^k\right)\hat{z}_{t+1}^{k\varphi}\quad$ is $N\left(0,\sigma_\varphi^2\right)$.   (17a)

With $\varrho_t^k = \varrho_t$ the functional form for the effects of monetary shocks on $\lambda^{k\varphi}(\varrho_t)$ is

$$\lambda^{k\varphi}(\varrho_t) = \frac{\lambda^{k\varphi}}{1 + e^{-\chi^\varrho(\log \varrho_t)}}. \tag{17b}$$

In (17b) the function is increasing in the size of monetary shocks; such shocks increase the effect of y_t^k on the perceived mean value of φ_{t+1}^k in (17), and this has

an important impact. Using an argument as in the lemma above, we can rationalize this belief by the condition

$$\left[\lambda^{k\varrho}(\varrho_t)\right]^2 \sigma_{\tilde{z}^k}^2 + \left[\sigma_{\tilde{z}^{k\varrho}}(\varrho_t)\right]^2 = \sigma_{\varrho}^2. \tag{17c}$$

This rationalization has four analogous implications to the time average, variance, and serial correlation of $\lambda^{k\varphi}(\varrho_t^k)y_t^k + \sigma_{\tilde{z}^{k\varphi}}(\varrho_t^k)\hat{z}_{t+1}^{k\varphi}$, and to its covariance with ϱ_t, which we do not repeat. Equations (17b) and (17c) determine $\sigma_{\tilde{z}^{k\varphi}}(\varrho_t^k)$ and restrict the feasible values of $\lambda^{k\varphi}$ in (17). As the process is hypothetical we set $\sigma_{\varphi}^2 = 1$, $\lambda^{k\varphi} = 1$ although these should be estimated from data.

We finally state the implication of assumption A.7, which says that agents use (11d) to forecast the effects of future states of belief. Consistency between (11d) and (12) requires

$$\sigma_{z^k} = \sigma_{y^k} \qquad \text{for } k = 1, 2. \tag{18}$$

3.4. Comments about the Computation Model and the Parameters

We will compute equilibria using a method of perturbation around the steady state of a riskless economy.[7] First, we make some comments to clarify the exposition.

3.4.1. STEADY STATE AND THE INITIAL ASSET DISTRIBUTION

(1) Given the growth context assumed, in order for the model to have a steady state, we assume $\gamma^1 = \gamma^2 = \gamma = 1.5$, $\beta_1 = \beta_2 = \beta = 0.99$, reflecting the quarterly model and $\zeta = 3.5$ to ensure that an agent's labor supply equals 0.2 in steady state. (2) Since the riskless steady state depends upon the initial distribution of assets, we assume the symmetry $\theta_0^1 = \theta_0^2 = \frac{1}{2}$, $b_0^1 = b_0^2 = 0$, $k_0^{kd} = \frac{1}{2}k^{d*}$, $n_0^{kM} = \frac{1}{2}n^{M*}$, $n_0^{kS} = \frac{1}{2}n^{S*}$. (3) It follows from (1) and (2) that the only model heterogeneity is the diversity of agents' beliefs.

3.4.2. ACCURACY OF THE EULER EQUATIONS

The model has nine state and twelve endogenous variables. To compute equilibria we expand policy functions to second-degree polynomials and compute 780 derivatives. This leads to a much higher precision than in most of the literature, based on log-linearization. A solution is acceptable only if the Euler equations are satisfied with a precision of 10^{-3} in the 5 percent neighborhood of the steady-state values of the aggregates (e.g., for values of capital k_t in the interval $0.95k^* \leq k_t \leq 1.05k^*$) and 10^{-2} in the 10 percent neighborhood of the steady state.

3.4.3. DEFINITION OF AN REE VERSUS AN RBE

Belief parameters of the agents are symmetric and the only asymmetry arises in the investment sector as between the two activities. Parameters are either based

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7. Details of the computational model are provided in an Appendix B that can be downloaded from the homepage of the first author at http://www.stanford.edu/~mordecai. The appendix covers some additional technical problems associated with the computational procedure.

on *actual econometric estimates* of the corresponding parameters in the economy or are hypothetical. Beliefs are characterized by the following parameters, which are the same for $k = 1, 2$:

$$\sigma_{z^k} = 0.8, \quad \lambda^{kv} = 0.002, \quad \lambda^{k\varrho} = 0.01, \quad \lambda^{k\varphi} = 1, \quad \rho_{yy} = 0.95, \quad \chi^\varrho = 20.$$

An REE is defined as an equilibrium where *agents believe the stationary measure is the truth* and is simply characterized by the conditions, for $k = 1, 2$,

$$\lambda^{kv} = \lambda^{k\varrho} = \lambda^{k\varphi} = 0, \quad \sigma_{z^k}^2 = 0, \quad \sigma_{z^k v}^2 = \sigma_v^2, \quad \sigma_{z^k \varrho}^2 = \sigma_\varrho^2,$$

$$\sigma_{z^k \varphi}^2 = \sigma_\varphi^2, \quad y_t^1 = y_t^2 = 0. \tag{19}$$

3.5. Simulation Results and What We Learn from Them

3.5.1. VOLATILITY CHARACTERISTICS

Table 2 reports standard deviations, first-order autocorrelations, and contemporaneous correlations with output for key variables in the REE and RBE simulations. All variables are log deviations from the model's deterministic trend. All data are HP filtered. We view the REE results as a reference to the volatility that would be generated by small, persistent exogenous productivity shocks. Indeed, with $\sigma_v = 0.002$ such shocks generate only a fraction of the observed volatility. In contrast, the table reveals that the level of volatility generated in the RBE is similar to the level observed in the data. For the United States the standard deviations of the variables are 1.81 for log g^N, 5.30 for log i^N, 1.31 for log c, and 1.79 for log L. These results exhibit the ability of beliefs in the RBE to propagate volatility endogenously in orders of magnitudes observed in the real data.

The two panels in Figure 2 exhibit time series of HP-filtered observations on log (GNP), log (investment). Since the level of volatility of log (GNP) is the same as in the U.S. data, the figure highlights the difference between the volatility generated by the exogenous shocks in comparison with the endogenous component generated by the beliefs of agents.

Table 2 also shows that the RBE fails to exhibit a correlation structure that is present in the real data and would be generated by a larger variance of the productivity shocks. This failure is instructive since it results from our assumptions and provides a deeper understanding of the propagation mechanism of the model. To explain it, we first observe that since agents seek to smooth consumption,

Table 2. Volatility Characteristics

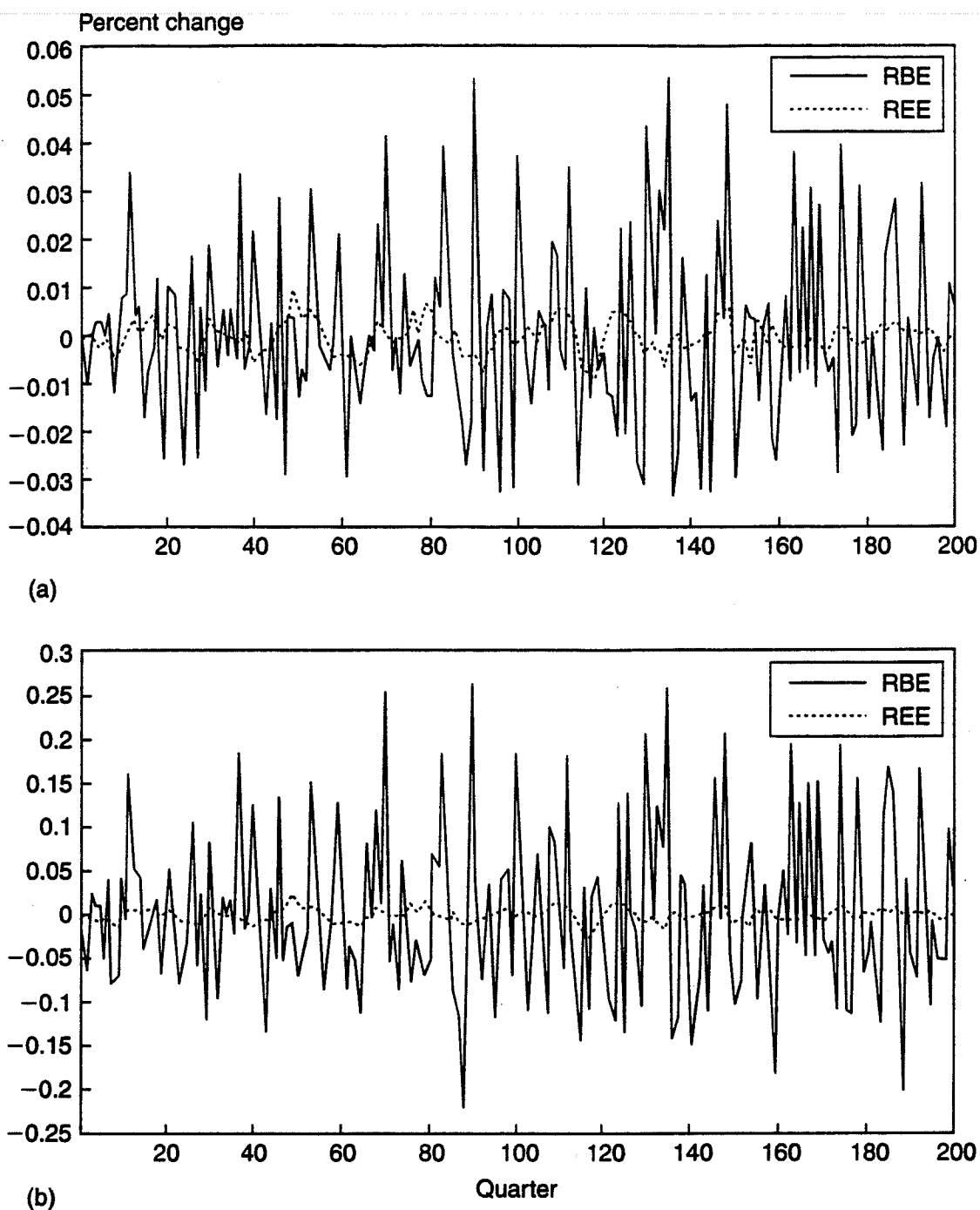|	Standard deviation		Relative standard deviation		First-order autocorrelation		Correlation with GNP	
	REE	RBE	REE	RBE	REE	RBE	REE	RBE
log g^N	0.33	1.80	1.00	1.00	0.72	−0.08	1.00	1.00
log i^N	0.79	9.08	2.39	5.04	0.71	−0.09	0.99	0.91
log c	0.18	1.28	0.55	0.71	0.74	−0.06	0.99	−0.30
log L	0.12	2.04	0.36	1.13	0.71	−0.07	0.98	0.30

Figure 2. Comparative volatility of GNP and investment in REE and RBE: (a) log (GNP), HP filtered; (b) log (investment), HP filtered.

their investment and consumption plans depend upon *their perception of the persistence of the random shocks.* High persistence in productivity shocks, which leads to persistence in the random component of the wage rate and the return on investments, leads to plans that associate higher investments with higher consumption and hence induce positive correlation among output, consumption, and investments. The fact is that the volatility of the RBE is generated by shocks in the investment goods sector and by the temporal behavior of the states of belief. We assume in (11c) that $\varphi_t = \rho_t^\varphi$, $\rho_t^\varphi \sim N(0, \sigma_\varphi^2)$ iid. In addition, the states of belief exhibit no persistence. Conditioning upon monetary shocks generates some

Table 3. Volume of Trade on Financial Markets

	REE	RBE
Mean TR	0	2.61
σ_{TR}	0	1.74
$\rho(TR_t, TR_{t-1})$	—	0.26
max TR_t	0	11.72
min TR_t	0	0.07

persistence, but the level is small and its effect is of second order. Hence, when agents believe that attractive investment opportunities are present, they perceive them to be temporary, with little persistence. These strong assumptions ensure small or negative serial correlation of the variables, and Table 2 demonstrates this fact. Also, without persistence, perceived high returns on investments provide a strong motive for agents to *substitute consumption for investments* and work harder to finance such projects. This explains the negative correlation between consumption and GNP and the negative correlation between consumption and investments. It also explains the low correlation between hours of work and GNP. For a calibrated RBE to exhibit the correlation structure observed in the data, the RBE would have to exhibit persistence in the rate of return on investments and/or in states of belief.[8]

3.5.2. VOLUME OF TRADE

Our economy has two financial assets: nominal bonds and ownership units in built capital goods. We thus make two observations. First, since the nominal debt instrument is of short duration, it matures each period and has to be traded again. Since bonds are in zero net supply, the volume of their trade is defined by $|q_t^b b_t^k|$. The volume of capital goods traded entails an accounting problem since old units of capital depreciate so the stock owned by agent k at t before trading is $(k_{t-1}^{kd} + i_{t-1}^k)(1 - \delta)/v^*$. The new amount that he buys is k_t^{kd}, so the value of his trade at date t is defined by $q_t^s\{k_t^{kd} - (k_{t-1}^{kd} + i_{t-1}^k)(1 - \delta)/v^*\}$. Owing to growth, a meaningful measure of the volume of trade is the value of trade relative to GNP. Thus we define the relative volume by

$$TR_t = \frac{1}{GNP_t}\left\{\left|q_t^s\left[k_t^{1d} - (k_{t-1}^{1d} + i_{t-1}^1)\frac{1-\delta}{v^*}\right]\right| + |q_t^b b_t^1|\right\}. \qquad (20)$$

Table 3 shows statistics based on 1,000 simulated observations on TR. The RBE generates a dramatic volume of trade and exhibits a large standard deviation of TR. Table 3 reveals that the extreme values are far apart, suggesting sharp variations in volume, which is a result that is consistent with those of Kurz and Motolese's

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8. To introduce persistence in the states of belief or in the φ_t process we would have to add state variables to a model whose equilibrium is already difficult to compute. We have elected not to do so since we view this work as a theoretical analysis that utilizes computation methods rather than a calibration effort.

(2001) on the dynamics of trade. Finally, the sum of money and bond holdings is large, with an average ratio to GNP of about 12.

3.5.3. MONEY NEUTRALITY AND THE PHILLIPS CURVE

Money is neutral in any REE of the model. An RBE also has an obvious money neutrality property, neutrality *on average,* which means that the average rate of inflation equals the average rate of increase of the money supply minus the average rate of growth of output. In all RBE the mean rate of increase in the money supply is v^*, which is the mean growth rate of output; hence the mean rate of inflation is 0. RBE are, however, generically money nonneutral.

To see that money is not neutral in an RBE we carry out a simple test. We first simulate the equilibrium assuming that both the technological shocks v_t as well as the money shocks ϱ_t are turned off and compute the statistics generated. Next we turn on the monetary shocks at the level of, say, $\sigma_\varrho = 0.05$, and recompute. Table 4 presents comparative results of this experiment. The fact that money injection has real effects is clear and the explanation for why money is not neutral is also simple. Monetary shocks have an effect on expectations of agents and although they observe the rate at which the money supply changes, their demand for money (which depends upon their beliefs) changes but not in the same way. The resulting percentage change in the price level is thus not equal to the percentage change in the money supply.

The analysis of the Phillips curve in an exact general equilibrium context is not a simple matter. It entails a relationship between two endogenous variables, both of which are functions defined by the equilibrium map (10), and this leaves no room for a good instrument to remove the bias. We simulated the model twice: first with $\sigma_\varrho > 0$, $\sigma_v > 0$ and second with $\sigma_\varrho > 0$, $\log v_t = 0$. We used samples of 5,000 observations and estimated the following simple regression model:

$$\log \pi_t = a_0 + a_1 \log g_t^N + a_2 \log \varrho_t + a_3 \log \pi_{t-1} + \vartheta_t$$

keeping in mind that $\log (g_t^N)$ is the log of the deviation of GNP from trend.

ESTIMATION FOR REE
1. With $\sigma_\varrho > 0$, $\sigma_v > 0$:

$$\log \pi_t = - \underset{(0.0001)}{0.003} - \underset{(0.0008)}{0.012} \log g_t^N + \underset{(0.001)}{0.999} \log \varrho_t - \underset{(0.0014)}{0.002} \log \pi_{t-1}$$

$$+ \vartheta_t, R^2 = 0.996.$$

Table 4. Money Nonneutrality of RBE (standard deviations in percent)

	$\log (\varrho_t) = 0, \log (v_t) = 0$	$\sigma_\rho = 0.05, \log (v_t) = 0$
$\log g^N$	1.93	2.05
$\log i^N$	9.38	10.12
$\log c$	1.38	2.18
$\log L$	2.11	3.26

2. With $\sigma_\varrho > 0$, $\log v_t = 0$:

$$\log \pi_t = \underset{(0.003)}{-0.004} + \underset{(0.000)}{0.000} \log g_t^N + \underset{(0.000)}{1.000} \log \varrho_t - \underset{(0.000)}{0.000} \log \pi_{t-1}$$
$$+ \vartheta_t, \; R^2 = 1.000.$$

ESTIMATION FOR RBE

1. With $\sigma_\varrho > 0$, $\sigma_v > 0$:

$$\log \pi_t = \underset{(0.004)}{-0.113} + \underset{(0.021)}{0.774} \log g_t^N + \underset{(0.028)}{1.414} \log \varrho_t - \underset{(0.011)}{0.445} \log \pi_{t-1}$$
$$+ \vartheta_t, \; R^2 = 0.47.$$

2. With $\sigma_\varrho > 0$, $\log v_t = 0$:

$$\log \pi_t = \underset{(0.004)}{-0.319} + \underset{(0.026)}{1.845} \log g_t^N + \underset{(0.022)}{1.419} \log \varrho_t - \underset{(0.008)}{0.468} \log \pi_{t-1}$$
$$+ \vartheta_t, \; R^2 = 0.67.$$

We have tried many different specifications and the conclusions are the same. The REE regressions show money neutrality. As for the RBE, in all regressions the Phillips curve coefficient of $\log g_t^N$ is large and positive.

3.5.4. IMPULSE RESPONSE TO MONETARY SHOCKS

Our final results relate to the impulse response functions of monetary shocks. The study of such functions reveals the manner in which the real effect of monetary shocks works through the RBE. A monetary shock in steady state is different from a productivity shock since a productivity shock forces the economy out of steady state through the *direct* first-order effects it has on output and resource prices. A monetary shock has only a second-order, *indirect* effect on the economy via its impact on parameters such as $\lambda^{kv}(\varrho_t^k)$, $\lambda^{k\varrho}(\varrho_t)$, and $\lambda^{k\varphi}(\varrho_t)$, which are *multiplied* by the states of belief y_t^k. To see the implication recall that every endogenous variable in the economy is represented by a polynomial whose derivatives we evaluate at the steady-state solution of the economy. Hence, the equilibrium is a perturbation of the steady state and the belief parameters (y_t^1, y_t^2) are represented in each polynomial as regular variables. The steady state of an RBE requires us to set $y_t^1 = 0$, $y_t^2 = 0$. Since in any impulse response function all variables are evaluated at steady state, to determine the impact of a monetary shock we evaluate this effect at steady state. An examination of the perception models shows that the impact of a monetary shock works through terms such as

$$\Upsilon_{t+1}^{kv} = \lambda^{kv}\left(\varrho_t^k\right) y_t^k + \sigma_{z^{kv}}\left(\varrho_t^k\right) \hat{z}_{t+1}^{kv}$$

in equations (13), (16), and (17). Indeed, this multiplicative form is the direct result of the rationality conditions: Additive terms would violate rationality. But now the answer is clear: At steady state we set $y_t^k = 0$ and all random variables are also set equal to zero so no monetary shock can have any effect. The conclusion is

that a monetary impulse of this RBE at the steady state has only a direct effect on inflation: A change in money supply causes a proportionate change in price level, leaving all real variables unchanged.

The picture changes drastically if we select an alternate state of belief $(y^{1*}, y^{2*}) \neq (0, 0)$ and hold it fixed. We now compute the long-term averages of equilibrium variables denoted by x^*, and create a new *hypothetical* steady state (y^{1*}, y^{2*}, x^*) for the economy. In an REE $(y^{1*}, y^{2*}, x^*) = (0, 0, x^*)$ for all (y^{1*}, y^{2*}) since states of belief do not matter. Having defined this steady state, we examine the impulse responses to a monetary shock. The interpretation of these responses is natural: They measure the impact of a monetary shock on real variables around (y^{1*}, y^{2*}, x^*). Since the risky economy is never at the $(y^{1*}, y^{2*}) = (0, 0)$ steady state, the magnitude of the impulse response varies with the point in space at which the economy is evaluated. Hence, the usual steady state with $(y^{1*}, y^{2*}) = (0, 0)$ has no significance and positions away from it better reflect the response of the risky economy to monetary shocks.

Based on the above considerations we selected $(y^{1*} = 0.4, y^{2*} = 0.4)$, which is half a standard deviation of the empirical distribution of (y_t^1, y_t^2), and study the response to an initial monetary shock of log $\varrho_t = 2$ percent. The results are presented in Figure 3. Some comments on these results:

1. *GNP:* This impulse response function exhibits the familiar hump-shaped form, known from empirical work on the effect of monetary shocks and hence is qualitatively in accord with standard results. Keep in mind, however, that this response function is generated entirely endogenously by the structure of market beliefs.

2. *Investment:* This function reveals the impact of the endogenous propagation mechanism of the model. The 2 percent monetary shock alters private expectations and causes a large 0.3 percent burst of investments that slowly decreases in magnitude. It is the size and sharp nature of this burst that explains the impulse response of other variables.

3. *Consumption:* Although the response of consumption exhibits the familiar hump-shaped form, it also has the feature of starting with a *negative* impulse at date 0. This is due to the special features of our model. The monetary shock alters expectations and results in a burst of investments. Owing to lack of persistence in perceived returns on investments, agents reduce present consumption in order to finance investments. After the initial decline, consumption rises, reaches a maximal deviation from trend, and then returns to trend.

4. *Nominal interest rate:* The model has no riskless asset so the impulse response function of the nominal interest rate is relatively simple. A 2 percent monetary shock causes only a 1.8 percent instantaneous inflation at date 0. The burst of investments leads to an increased demand for funds, which pushes real interest rates higher. As the investment effect declines, the full price level adjustment is completed.

5. *Hours:* This function shows the final dimension of the increased demand for investments generated by a monetary shock. As agents perceive
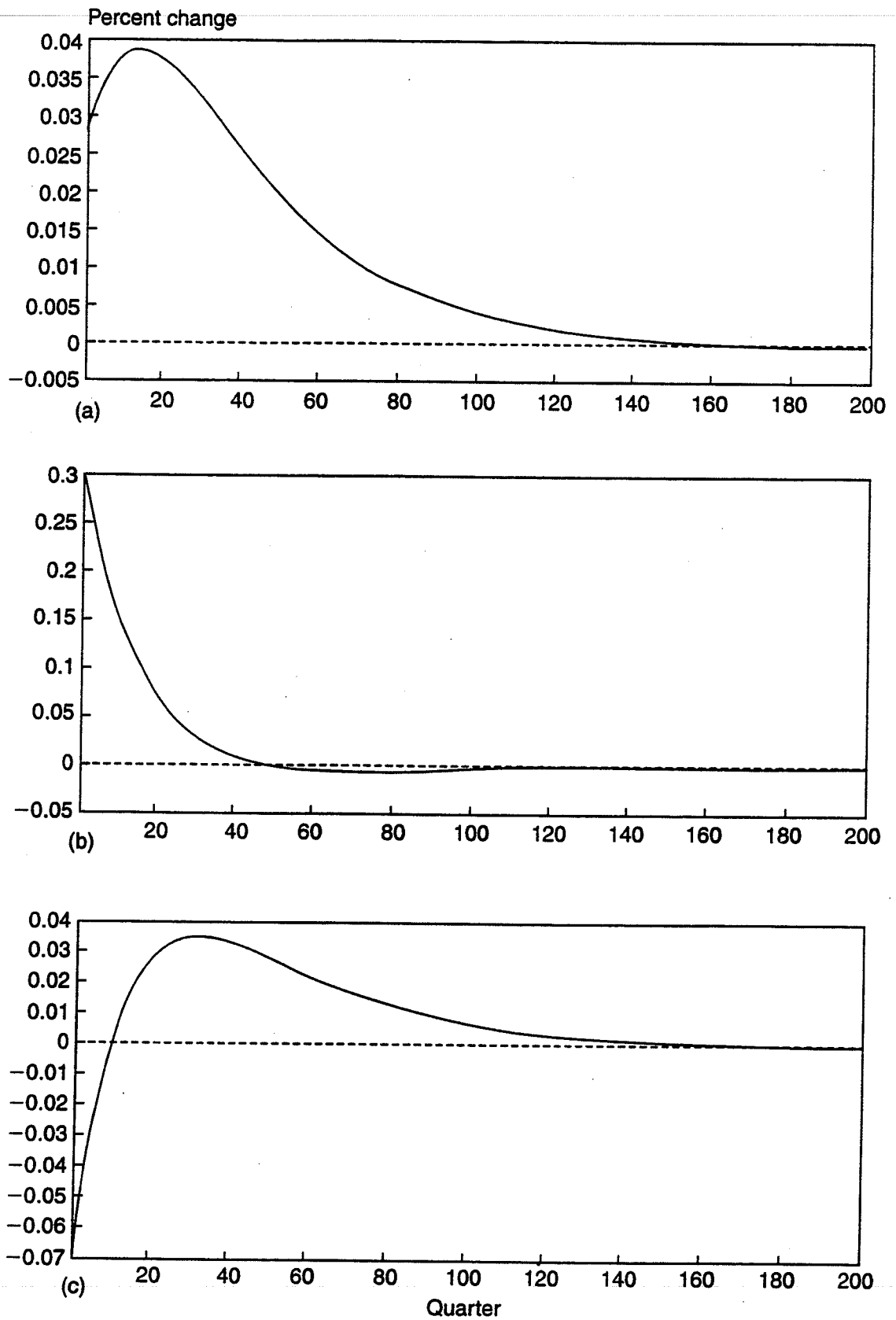
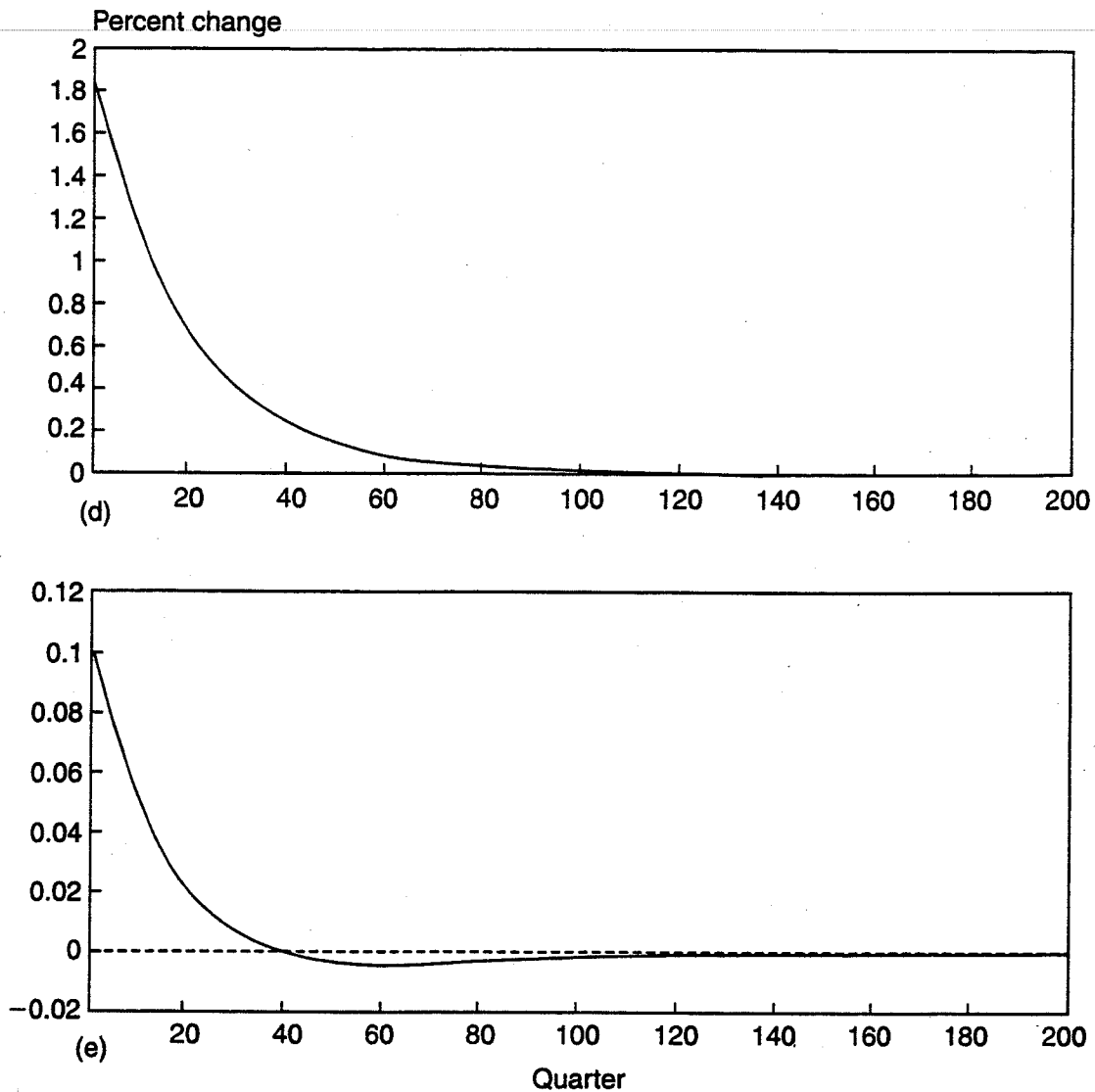*Figure 3.* Impulse response to a +2 percent monetary shock: (a) log(GNP); (b) log(investment); (c) log(consumption).

Percent change

(d)

(e)

Quarter

Figure 3. (*Continued*) (d) nominal interest rate; (e) log (hours).

higher rates of return on investments they desire to work more hours to finance such investments, revealing a 0.1 percent rise in hours.

3.5.5. Why Monetary Shocks Have Real Effects and Positive Shocks Increase Output: A Simple Intuitive Explanation

Why do monetary shocks have real effects and why do positive shocks have positive effects on output? These are two *separate* questions that we address separately. The fact that money has a real effect in an RBE is a *generic property arising from the diversity of beliefs*. Monetary shocks have a real effect because agents have diverse forecasts of all variables in the economy. They disagree about the effects of monetary shocks on inflation because they have different forecasts of the rate of return on investments and hence of the demand for money that results from differing investment rates. Also, agents disagree about the distribution of future monetary shocks and hence about the cost of holding money. This argument is not reduced to a sunspot argument saying that money has an effect because agents

expect it to have an effect. Agents have different forecasts of the real variables in the economy and once they disagree on the inflation forecast given any monetary shock, monetary shocks have real effects. This is analogous to Lucas's (1972) argument explaining nonneutrality by agents' "confusion" between monetary and real shocks. In an RBE there is no confusion about shocks, only disagreement about forecasting the future, and this is sufficient for money to have real effects (for a similar argument see Motolese, 2000, 2001). Once agents perceive money to have a real effect, it is rational for them to incorporate monetary shocks as a component affecting their own beliefs about all variables in the economy (i.e., for ϱ_t to affect agents' perceptions). We now turn to the second question.

Why do *positive* monetary shocks *increase* output? Given the fact, explained in the previous paragraph, that money has an effect on the belief of agents, the answer to this second question is simple: A positive monetary shock in the model generates optimism among agents about higher rates of return on investments and thus causes them to be willing to engage in more risky investments. When this occurs they increase their inputs into investment projects. For that purpose they work harder and lower their consumption a bit in order to finance the desired investments. In our model these effects are short lived because there is no persistence in either the beliefs or in the investment opportunities. However, the fact that these effects are short lived has nothing to do with the idea of agents being "surprised" by monetary shocks. Our explanation is typical of the mechanism that operates in any RBE model. In future work we hope to modify the model so that this conclusion becomes more transparent.

In the present model the mechanism is a bit complicated because of our assumption that there are two investment opportunities and only one iid random variable to determine their productivity. The result is that whenever agents are optimistic about higher returns in project S they *must* be pessimistic about returns in project M. Hence, it is impossible for them to be optimistic about future returns in both projects. When $y_t^k > 0$ they are optimistic about project M, and they increase investments in M but decrease investments in S. The opposite is true when $y_t^k < 0$. In the model monetary shocks alter the impact of y_t^k on the perceptions of future returns on assets. Hence, in order to stipulate that agents take more risk in response to a monetary shock we require that $\left|\lambda^{k\varphi}(\varrho_t)\right|$ in equation (17c) *increases* with ϱ_t. This achieves the objective because when $\left|\lambda^{k\varphi}(\varrho_t)\right|$ increases with ϱ_t, a larger monetary shock amplifies the impact of y_t^k on forecasted returns on investment. Indeed, it increases the forecasted deviation of φ_{t+1} from 0 and hence increases the expected returns on investments. This, in turn, increases the desire to invest.

But then, one may object, our conclusion regarding the positive effect of monetary shocks hinges on the shocks having the "right" effect on beliefs [via a parameter such as $\lambda^{k\varphi}(\varrho_t)$]. This is entirely true: The positive association will not be present unless parameters such as $\left|\lambda^{k\varphi}(\varrho_t)\right|$ change in the "right" way. But then recall that in an RBE monetary shocks have real effects and all agents recognize this. Once real effects—positive or negative—are present in the economy then, like any endogenous uncertainty, they become part of reality and impact the beliefs of agents. In fact, the idea that actions of the central bank affect agents' expectations is central to modern thinking about monetary policy. Most discussions about

credibility of the central bank have to do with the impact of policy on beliefs, and the effect of policy on beliefs is central to the DNK theory discussed earlier (e.g., Woodford, 2001b).

The fact that policy has an impact on beliefs is clear; the only question is the nature of that impact. In this chapter we have not developed a formal theory of this impact. Instead, we assumed that such an effect exists *and have proved that it can be rationalized.* The assumption that parameters such as $\left|\lambda^{k\varphi}(\varrho_t)\right|$ have the right slope is then a condition specifying what it takes for a monetary shock to have a positive effect on output. This discussion also shows that a monetary equilibrium cannot be complete without a full specification of monetary policy.

3.5.6. WHAT HAVE WE LEARNED FROM THE MODEL?

Computational economics is a useful tool for theoretical work in dynamic equilibrium analysis when analytical results are difficult to obtain. Many of our numerical results will change with changes in parameters or specifications. Our model has several unrealistic features that result in no persistence of investment opportunities and no persistence of beliefs. Also, the linear structure of the investment sector is too simplistic and results in low volatility of the price of traded capital goods. Hence, any effort to calibrate the model would require significant model restructuring. However, our objective was to formulate an analytical framework in which the impact of diverse beliefs can be evaluated and, in our view, the present model demonstrated the importance of this economic factor. There are other conclusions that, we believe, can be drawn at this time since *they are generic to any RBE model.* We offer a short summary of these:

1. Diversity of beliefs in an RBE is a propagation mechanism capable of producing volatility of the order of magnitude observed in the data. The mechanism operating in the investment sector applies to other intertemporal decisions such as consumer durables and public investments.
2. The RBE offers an integrated theory of real and financial volatility, where fluctuations in both sectors are *jointly* propagated. It provides a direct explanation for the high volume of trade on financial markets and for the high volatility of aggregate investments.
3. Diverse beliefs in the RBE explain most of the familiar features of monetary equilibria in which money has real effects. This includes money nonneutrality and Phillips curve and impulse response functions with respect to monetary shocks but these have nothing to do with "surprises" of agents. It arises in the RBE owing to the fact that a positive monetary shock reduces the risk premium that agents require in order to engage in risky investments.

A methodological point is also appropriate. We use the "distribution of beliefs" as an *explanatory variable* of market performance. Indeed, the use of the distribution of beliefs as an explanatory variable is central to all work on market volatility in RBE cited in this chapter. It is our view that the distribution of beliefs is as valid an explanatory variable as any information about characteristics of agents or technology used routinely in economic analysis to explain market performance.

For this reason we think that the distribution of beliefs is useful information for the formulation of monetary policy. That is, since an important component of economic fluctuations is endogenous, caused by the beliefs of agents, the conduct of monetary policy could be improved by incorporating information about the distribution of beliefs in the market.

4. WHAT IS THE RATIONALE FOR AND OBJECT OF MONETARY POLICY?

We turn now to a brief discussion of the justification for monetary policy. Owing to the endogenous component of volatility, the level of risk in a market economy is greater than the level induced by exogenous shocks. But if social risk is caused by human conduct, then society may elect to place some limits on individual choice so as to reduce the level of such endogenous volatility. This was a line of thought discussed by Kurz (1997a, pp. ix–x) and explored by Nielsen (1998, 2000). It is a fact that an RBE with complete hedging opportunities is ex ante Pareto-optimal and in that case any effective public policy will make some agents worse off in terms of their *ex ante expected utility*. As a side remark we note that RBE are generically incomplete owing to the lack of observability of the states of beliefs, as is the case with the model economy of this chapter. Nevertheless, the theory of RBE offers an important role for the central bank *regardless of market completeness*. The aim of monetary policy is to reduce the excess endogenous volatility component of economic activity and of the price level. Since a central bank cannot reduce volatility caused by exogenous shocks, the task of reducing the endogenous component is both attainable and socially desirable. However, this task requires some choices.

Investors' freedom of choice is the foundation of all efficiency considerations leading to ex ante Pareto optimality of the allocation in a market economy. However, the price of such freedom is the excessive level of volatility in the market, which is caused by their diverse and often inconsistent beliefs. To support our position that monetary policy is justified in an RBE, we have to argue that the ex post excess volatility of a market economy is undesirable.

The problem of reconciling ex ante and ex post outcomes is not new. It was discussed by Diamond (1967), Drèze (1970), Starr (1973), Mirrlees (1974), Hammond (1981, 1983), Nielsen (1998, 2000), Silvestre (2002), and others. The interest in an ex post concept of optimality was motivated by two considerations. First, researchers held the view that in an uncertain world agents may hold "incorrect" probability beliefs and hence regret their decisions (e.g., Hammond, 1981, p. 236). Also, social preferences over income distribution could be sensitive to ex post outcomes rather than ex ante anticipated distribution of consumptions. Second, diverse and often inconsistent probability beliefs raise difficulties in defining a "representative consumer" who holds a social expected utility.[9] Hammond's (1981,

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9. Diverse and inconsistent subjective probability beliefs raise difficulties in defining an equilibrium in games with incomplete information and have led to the "Harsanyi doctrine" of a common prior in game theory.

1983) concept of ex post welfare function proposes that society should disregard the probability beliefs of individual agents in favor of the social planner's own consistent probabilities over states.

The RBE theory offers some new elements that enable a different perspective of ex post optimality. In an RBE agents typically hold diverse nonstationary beliefs and society, represented by the central bank, recognizes that when agents hold diverse and inconsistent beliefs *some or all* of them are wrong. Since society does not have better information than private agents, the central bank cannot determine whose beliefs are right. Hence, monetary policy must be symmetric with respect to the diverse beliefs. Over time any agent may hold correct or incorrect beliefs in the form of a subjective joint distribution over observables. Hence, public policy must be optimal in the long run, in the sense that it should be a good policy for any configuration of beliefs that agents may hold over time. The rationality conditions in an RBE imply that the mean belief of any agent over time is exactly the stationary measure,[10] and this probability is also the expected probability belief of an agent on any date. Hence, public policy should be optimal with respect to the stationary measure. That is, it should seek to maximize expected welfare not with respect to the belief of any particular agent but rather with respect to the long-run empirical distribution. Also, agents agree on the stationary measure since it is the probability on sequences deduced from the empirical distribution of observables, and this distribution is known to all. An example will illustrate.

An Example. For simplicity, this example focuses on improving long-run welfare rather than on monetary policy. To that end, consider an infinite horizon economy with a complete market structure (an assumption not satisfied in the RBE above). Within this economy there is a leading sector using a technology that is similar to the one used in the investment sector of the model in Section 3. There are a large number of identical firms each led by an investor-entrepreneur. At each date an investor-entrepreneur uses the publicly available technology to produce output. The constant-returns-to-scale technology employs two agents who live for one period: the investor with an endowment of 1 and a worker who works for the investor and receives 25 percent of the output.

At every date t an investor allocates his endowment between activities M and S, whose technological nature varies over time. However, at all dates uncertainty is represented by two states, L and R. The long-term empirical distribution of the realized states is $\frac{1}{2}$ for L and $\frac{1}{2}$ for R with no serial correlation, and hence the stationary measure is iid with probability of each state being $\frac{1}{2}$. Trading and investments take place in the morning and output and consumption in the afternoon. Investors' beliefs are such that at each t they are certain that either L or R will be realized. They hold rational beliefs so half of the time they are certain L will be realized and half of the time they are certain R will be realized. Beliefs of investors are perfectly correlated. Output possibilities are represented by the

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10. That is, the mean transition function or joint density over observable variables employed by an agent at different dates is equal to the transition function or joint density over all observable variables in the economy defined by the stationary measure (or, empirical distribution).

following function. If an investor at t places X units in activity S and Y units in activity M subject to $X + Y = 1$, output will be the following random variable:

$$\text{Output}_t \equiv \begin{cases} 2.8X + 1.8Y & \text{if} \quad L \text{ is realized} \\ 2Y & \text{if} \quad R \text{ is realized.} \end{cases}$$

When an investor is certain L will be realized he selects $(X = 1, Y = 0)$; when he is certain R will be realized he selects $(X = 0, Y = 1)$. Since the stationary measure is iid with the probability of L being $\frac{1}{2}$, the empirical distribution of outputs and individual shares of the two participants are:

Proportion of time	Output	Investor's share	Worker's share
25%	2.800	2.100	0.700
25%	0	0	0
25%	2.000	1.500	0.500
25%	1.800	1.350	0.450
Mean:	1.650	1.2375	0.4125

The "proportion of time" entries in the table are probabilities according to the stationary measure. Note that the free market allocation results in outstanding performance 75 percent of the time. However, in 25 percent of the time it has catastrophic consequences, generating significant volatility. Now if society selects $X = \frac{1}{15}$ and $Y = \frac{14}{15}$ at all dates, the following allocation is socially feasible:

Proportion of time	Output	Investor's share	Worker's share
25%	1.867	1.400	0.467
25%	1.867	1.400	0.467
25%	1.867	1.400	0.467
25%	1.867	1.400	0.467
Mean:	1.867	1.400	0.467

Although the first allocation is ex ante Pareto optimal *relative to the beliefs of agents*, the second allocation stochastically dominates the first relative to the stationary measure. A social planner with concave utility will prefer the second allocation if he uses the iid stationary measure.

In our view the second allocation is an appropriate goal for stabilization policy. One may argue that in the free market allocation agents have opportunities to hedge their positions and ensure against all risks, given their beliefs. This is entirely true but fails to address the fact that the first allocation results both in excess fluctuations as well as in a significant loss of social resources in the catastrophic states. Moreover, the bad states, which entail the loss of social resources, are endogenous, man-made, and subject incentives. They are not due to exogenous factors but rather to beliefs and decisions of agents. Because of these lost resources the long-term average consumption is higher in the second allocation than in

the first. The example is easily extended to other forms of investment. Thus, the problem of economic volatility is not only one of smoothing consumption. It also entails lost social resources resulting from intertemporal decisions that are optimal relative to the agent's wrong beliefs, but which may not be socially desirable in the long run. Some may regard our example as one of market externalities of beliefs and there is some truth to this label. However, this is merely a terminological resolution. In future work we shall explore these issues in more detail.

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